Magnetic Field Mapping - Full Procedure

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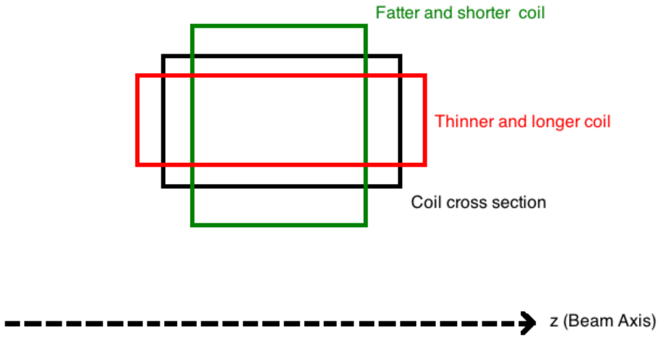
Geometrical Fit

The geometrical fit aims to account for most of the field.
Fit two axial ‘bracketing fields’ which have different aspect ratios compared to the coil dimensions, to the data by mixing them.



Geometrical Fit Procedure

A brief overview of the fitting procedure:

- Mix bracketing fields (f1 & f2) in proportion p

$$B^{\text{mix}}_{r,\phi,z} = pB^{f1}_{r,\phi,z} + (p-1)B^{f2}_{r,\phi,z}$$

- Scale the resultant field by f_B. Also scale coordinates by f_c

$$B^{scaled}_{r,\phi,z} = f_B B_{r,\phi,z}$$

$$r_{scaled} = f_c r, \quad z_{scaled} = f_c z$$

- Convert field into Cartesian coordinates
- Rotate the field components and coordinates about the x and y axis by θ_x, θ_y

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ \sin\theta_y \sin\theta_x & \cos\theta_x & -\cos\theta_y \sin\theta_x \\ -\sin\theta_y \cos\theta_x & \sin\theta_x & \cos\theta_y \cos\theta_x \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} B_{x,new} \\ B_{y,new} \\ B_{z,new} \end{bmatrix} = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ \sin\theta_y \sin\theta_x & \cos\theta_x & -\cos\theta_y \sin\theta_x \\ -\sin\theta_y \cos\theta_x & \sin\theta_x & \cos\theta_y \cos\theta_x \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

- Add x, y and z offsets

$$x_{new} = x + p_x, \quad y_{new} = y + p_y, \quad z_{new} = z + p_z$$

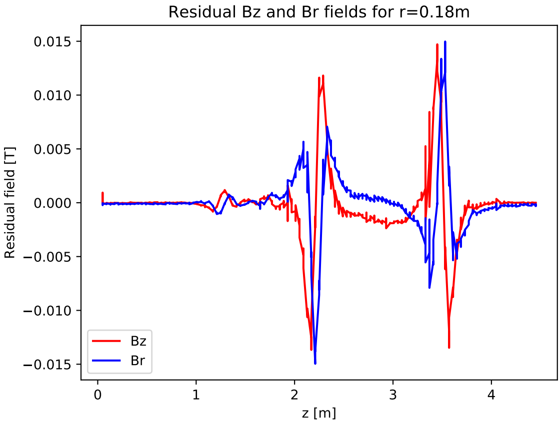
- Interpolate rotated field onto the same (polar) coordinates as the data being fitted to
- Calculate χ^2

$$\chi^2 = \sum_i^{\text{N datapoints}} \left(\frac{B_z^{data} - B_z^{model}}{\sigma} \right)^2 + \sum_i^{\text{N datapoints}} \left(\frac{B_r^{data} - B_r^{model}}{\sigma} \right)^2$$

Minuit (migrad) is used to find the minimum χ^2 using the free parameters:
p, f_B, f_c, θ_x, θ_y, p_x, p_y, p_z

Note that σ is the error on the hall probes which is difficult to pinpoint what it actually is. The presentation from the CERN mapping team gave $\sigma = 20$G, so that is what is used although it's not too important

Some Results



Residual field between the geometrical fit for SSD Centre Coil and the data.

Bonus: Magnetised Iron

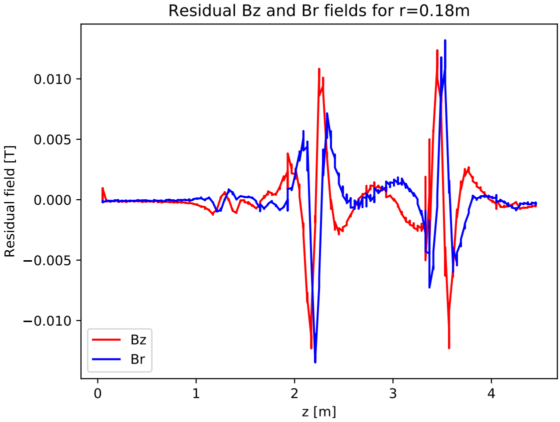
Also possible to add some Fourier-Bessel terms to take into account any magnetised iron around the coils.

$$B_z = \sum_{n=1}^{4} C_n \cos\left(\frac{nz}{L}\right) I_0\left(\frac{nr}{L}\right)$$

$$B_r = \sum_{n=1}^{4} C_n \sin\left(\frac{nz}{L}\right) I_1\left(\frac{nr}{L}\right)$$

L is 1m for End and Match coils and 1.5m for Centre coils and I_0, I_1 are modified bessel functions.

Adds 4 more free parameters to the geometrical fit.



Same plot as before but with FB terms for magnetised iron.

Fourier-Bessel Fit

As seen in previous plots, the geometrical fit doesn't fit the field perfectly. There are still features in the data that could be caused by various things, variations in the coil winding density for example.

- The geometrical fit fields obeys Maxwell's equation $\nabla \times B = 0$
- It is sensible to assume the residual field also obeys this equation
- Use a Fourier-Bessel series that obeys Maxwells equations to fit to the residual field
- Residual field is given by: $B_{\text{resid}} = B_{\text{data}} - B_{\text{geofit}}$
- FB field fitted to represent residual field. So the full model is then:

$$B_{\text{model}} = B_{\text{geofit}} + B_{\text{FB}} \approx B_{\text{data}}$$

$$
\begin{aligned}
B_z(r,\phi,z) &= \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} A_{nl}\mathrm{I}_n\left(\frac{l\pi}{z_{\max}}r\right)\cos(n\phi+\alpha_{nl})\cos\left(\frac{l\pi}{z_{\max}}z\right) \\
&- \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} B_{nl}\mathrm{I}_n\left(\frac{l\pi}{z_{\max}}r\right)\cos(n\phi+\beta_{nl})\sin\left(\frac{l\pi}{z_{\max}}z\right) \\
&+ \sum_{n=0}^{\infty} A_{n0}r^n\cos(n\phi+\alpha_{n0}) \\
&+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm}\mathrm{J}_n\left(\frac{\zeta_{nm}}{r_{\max}}r\right)\cos(n\phi+\gamma_{nm})\cosh\left(\frac{\zeta_{nm}}{r_{\max}}z\right) \\
&+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} D_{nm}\mathrm{J}_n\left(\frac{\zeta_{nm}}{r_{\max}}r\right)\cos(n\phi+\delta_{nm})\sinh\left(\frac{\zeta_{nm}}{r_{\max}}z\right)
\end{aligned}
$$

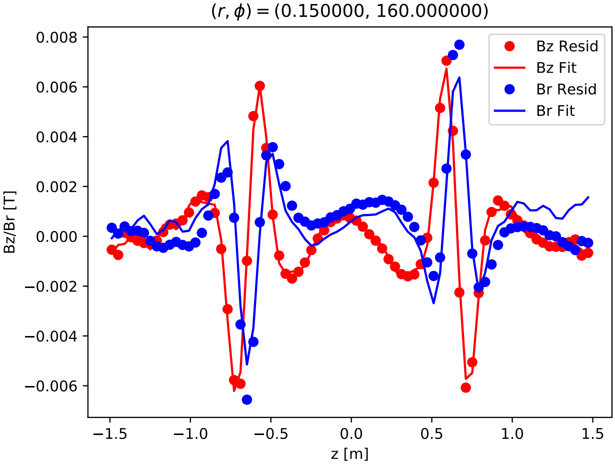
Let r_{max} be the radius of the outermost hall probe. However in practice the second outermost hall probe is used since the outermost has to be taken off for parts of the field map to be performed.

- Terms with C and D coefficients (hyperbolic terms) are equal to 0 at this radius due to the nature of the Bessel functions
- Use minuit to find $A_{nl}, \alpha_{nl}, B_{nl}, \beta_{nl}$ coefficients and phases by fitting to the cylindrical surface of measurements
- Then to find the hyperbolic term coefficients, fit to all data points on the two ends of the cylinder of measurements after subtracting the field contribution due to the Fourier terms

$$
\begin{aligned}
B_r(r,\phi,z) &= \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} A_{nl}\mathrm{I}'_n\left(\frac{l\pi}{z_{\max}}r\right)\cos(n\phi+\alpha_{nl})\sin\left(\frac{l\pi}{z_{\max}}z\right) \\
&+ \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} B_{nl}\mathrm{I}'_n\left(\frac{l\pi}{z_{\max}}r\right)\cos(n\phi+\beta_{nl})\cos\left(\frac{l\pi}{z_{\max}}z\right) \\
&+ \sum_{n=0}^{\infty} A_{n0}nr^{n-1}\cos(n\phi+\alpha_{n0})z \\
&+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm}\mathrm{J}'_n\left(\frac{\zeta_{nm}}{r_{\max}}r\right)\cos(n\phi+\gamma_{nm})\sinh\left(\frac{\zeta_{nm}}{r_{\max}}z\right) \\
&+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} D_{nm}\mathrm{J}'_n\left(\frac{\zeta_{nm}}{r_{\max}}r\right)\cos(n\phi+\delta_{nm})\cosh\left(\frac{\zeta_{nm}}{r_{\max}}z\right) \\
&+ \sum_{n=0}^{\infty} E_n nr^{n-1}\cos(n\phi+\varepsilon_n)
\end{aligned}
$$

- B_r and B_ϕ have an extra term, known as the multipole term, with coefficients E_n and phases ϵ_n.
- To find these, subtract contributions due to the Fourier and Hyperbolic terms from B_r on the cylinder surface.
- For each ϕ, average over z (multipole terms do not depend on z)
- Fit to find coefficients and phases

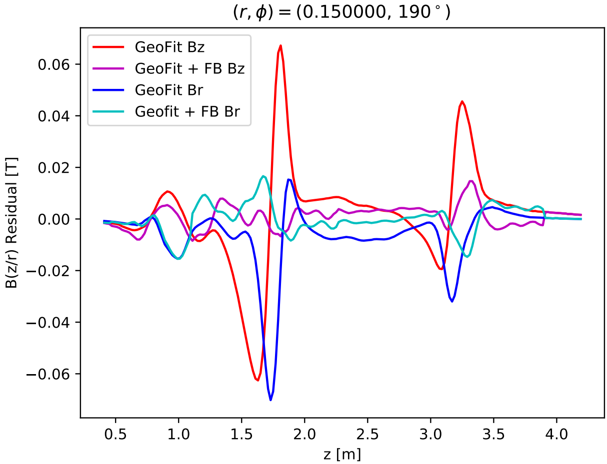
FB results

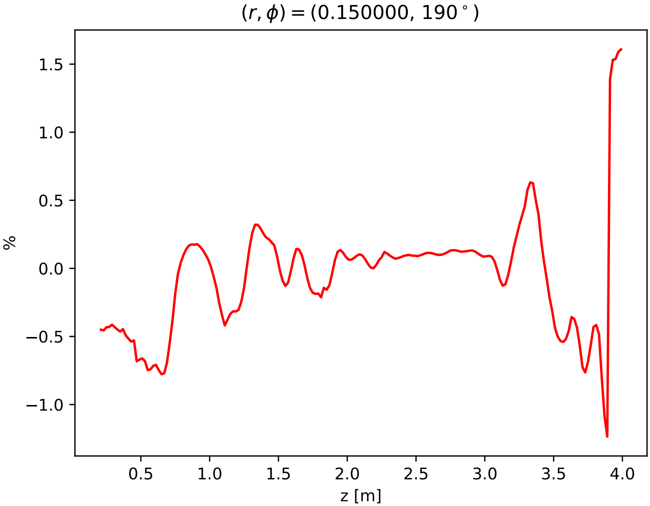


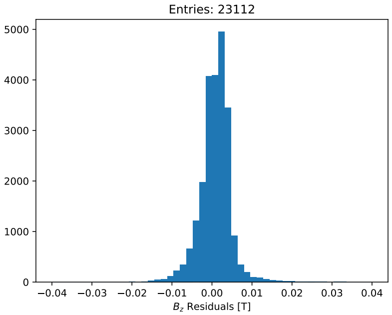
FB fit to SSD Centre coil residual field. Number of terms used ≈ 100

Some results of full model

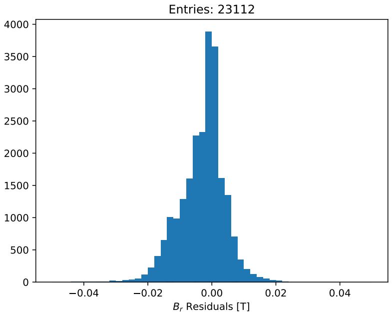
Now for some results of the whole model together for SSU Solenoid mode $I_{CC} = 219.2\text{A}$




(r, ϕ) = (0.150000, 190 °)
1.5
1.0
0.5
0.0
−0.5
−1.0
%
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
z [m]



Entries: 23112
5000
4000
3000
2000
1000
0
−0.04
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03
0.04
B_z Residuals [T]



Entries: 23112
4000
3500
3000
2500
2000
1500
1000
500
0
−0.04
−0.02
0.00
0.02
0.04
B_r Residuals [T]

Any Questions?