Errors

PION RUN: Example (KL cut = 4000):

N_μ^{tot}, number of muons before KL cut $N_\mu^{tot} = 53333$
N_μ^{cut}, number of muons that pass KL cut $N_\mu^{cut} = 95$
N_π^{tot}, number of pions before KL cut $N_\pi^{tot} = 68928$
N_π^{cut}, number of pions that pass KL cut $N_\pi^{cut} = 157$

$\sigma(N_\mu^{tot}) = \sqrt{N_\mu^{tot}}$, uncertainty of N_μ^{tot} $\sigma(N_\mu^{tot}) = 230.9$
$\sigma(N_\mu^{cut}) = \sqrt{N_\mu^{cut}}$, uncertainty of N_μ^{cut} $\sigma(N_\mu^{cut}) = 9.7$
$\sigma(N_\pi^{tot}) = \sqrt{N_\pi^{tot}}$, uncertainty of N_π^{tot} $\sigma(N_\pi^{tot}) = 262.5$
$\sigma(N_\pi^{cut}) = \sqrt{N_\pi^{cut}}$, uncertainty of N_π^{cut} $\sigma(N_\pi^{cut}) = 12.5$

$k_\mu = \frac{N_\mu^{cut}}{N_\mu^{tot}}$, muon fraction $k_\mu = 0.0018$
$k_\pi = \frac{N_\pi^{cut}}{N_\pi^{tot}}$, pion fraction $k_\pi = 0.0750$

$\sigma(k_\mu)$, uncertainty of k_μ
$\sigma(k_\pi)$, uncertainty of k_π

Since N_μ^{cut} and N_μ^{tot} are correlated, we can not simply find derivatives.
Instead we go to uncorellated N_μ^{cut} and N_μ^{fail}, the later are events that do not pass the cut.
Then one can find derivative of

$k_\mu = \frac{N_\mu^{cut}}{N_\mu^{cut}+N_\mu^{fail}}$, which is

$\sigma(k_\mu)^2 = (\frac{dk_\mu}{dN_\mu^{cut}})^2(\sigma(N_\mu^{cut}))^2 + (\frac{dk_\mu}{dN_\mu^{fail}})^2(\sigma(N_\mu^{fail}))^2 = \frac{k_\mu(1-k_\mu)}{N_\mu^{tot}}$ $\sigma(k_\mu) = 0.0002$

and similar

$\sigma(k_\pi)^2 = \frac{k_\pi(1-k_\pi)}{N_\pi^{tot}}$ $\sigma(k_\pi) = 0.0010$

MUON RUN:

R_μ, number of muons in sample $R_\mu = ?$
R_π, number of pions in sample $R_\pi = ?$
R^{tot}, number of particles before KL cut $R^{tot} = 72710$
R^{cut}, number of particles after KL cut $R^{cut} = 157$

$\sigma(R_\mu)$, uncertainty of R_μ $\sigma(R_\mu) = ?$
$\sigma(R_\pi)$, uncertainty of R_π $\sigma(R_\pi) = ?$
$\sigma(R^{tot}) = \sqrt{R^{tot}}$, uncertainty of R^{tot} $\sigma(R^{tot}) = 269.6$
$\sigma(R^{cut}) = \sqrt{R^{cut}}$, uncertainty of R^{cut} $\sigma(R^{cut}) = 12.5$

$R^{tot} = R_\mu + R_\pi$
$R^{cut} = k_\mu R_\mu + k_\pi R_\pi$

$\Rightarrow$

$R_\pi = \frac{(R^{cut} - k_\mu R^{tot})}{k_\pi - k_\mu}$ $R_\pi = 375.3$

$R_\mu = R^{tot} - R_\pi$ $R_\mu = 72334.7$

Now the derivatives of R_π are:

$\frac{dR_\pi}{dR^{cut}} = \frac{1}{k_\pi - k_\mu}$
$\frac{dR_\pi}{dR^{tot}} = -\frac{k_\mu}{k_\pi - k_\mu}$
$\frac{dR_\pi}{dk_\pi} = -\frac{R^{cut} - k_\mu R^{tot}}{(k_\pi - k_\mu)^2}$
$\frac{dR_\pi}{dk_\mu} = \frac{R^{cut} - k_\pi R^{tot}}{(k_\pi - k_\mu)^2}$

And the uncertainty $\sigma(R_\pi)$ is

$\sigma(R_\pi)^2 = (\frac{dR_\pi}{dR^{cut}})^2(\sigma(R^{cut}))^2 + (\frac{dR_\pi}{dR^{tot}})^2(\sigma(R^{tot}))^2 + (\frac{dR_\pi}{dk_\pi})^2(\sigma(k_\pi))^2 + (\frac{dR_\pi}{dk_\mu})^2(\sigma(k_\mu))^2$

. $\sigma(R_\pi) = 366.8$

and

$\sigma(R_\mu)^2 = \sigma(R^{tot})^2 + \sigma(R_\pi)^2$ $\sigma(R_\mu) = 248.7$

$q_\mu = \frac{R_\mu}{R^{tot}}$, muon fraction in muon run $q_\mu = 0.9948$
$q_\pi = \frac{R_\pi}{R^{tot}}$, pion fraction in muon run $q_\pi = 0.0052$

$\sigma(q_\mu)$, uncertainty of q_μ
$\sigma(q_\pi)$, uncertainty of q_π, where again due to correlation

$q_\mu = \frac{R_\mu}{R_\mu + R_\pi}$, so

$\sigma(q_\mu)^2 = (\frac{dq_\mu}{dR_\mu})^2(\sigma(R_\mu))^2 + (\frac{dq_\mu}{dR_\pi})^2(\sigma(R_\pi))^2$, where

$\frac{dq_\mu}{dR_\mu} = \frac{q_\pi}{R^{tot}}$, and
$\frac{dq_\mu}{dR_\pi} = -\frac{q_\mu}{R^{tot}}$ $\sigma(q_\mu) = 0.0050$

similar for $\sigma(q_\pi)^2$ $\sigma(q_\pi) = 0.0034$

Few numbers

Table 1: Pion and muon fractions in nominal muon run for different KL cuts

KL cut	2500	4000	5500	7000
N_μ^{tot}	53333	53333	53333	53333
N_μ^{cut}	485	95	24	6
N_π^{tot}	68928	68928	68928	68928
N_π^{cut}	9093	5171	2722	1357
k_μ, %	0.91±0.04	0.18±0.02	0.05±0.00	0.01±0.00
k_π %	13.19±0.13	7.50±0.10	3.95±0.07	1.97±0.05
R^{tot}	72710	72710	72710	72710
R^{cut}	848	157	41	15
R_μ	71189.2	72334.7	72497.9	72361.6
R_π	1520.8	375.3	212.1	348.4
q_μ %	97.91±0.59	99.48±0.50	99.71±0.49	99.52±0.51
q_π %	2.1±0.46	0.52±0.34	0.29±0.33	0.48±0.36