Momentum Measurement From TOFs

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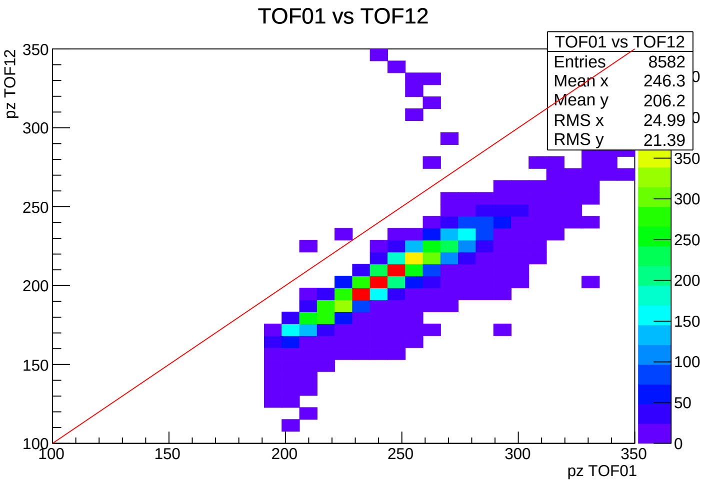
Naive Momentum Calculation

$$p = \frac{mc}{\sqrt{\left(\frac{c\Delta t}{\Delta s}\right)^2 - 1}} \tag{1}$$

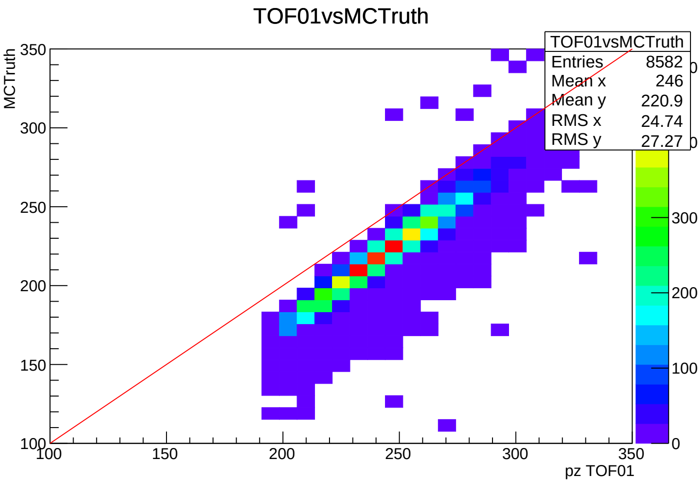
- In previous iterations of the analysis formula 1 has been used to calculate the momentum with the TOFs providing Δt and where Δs is known from the surveys
- The formula makes several assumptions namely
  - μ are on axis and travel in a straight line
  - $dE/dx = 0$
- To correct for this a statistical correction based on the Monte Carlo was proposed by Ryan
  - Not entirely satisfactory, an analytic expression describing the μ momentum was requested

Naive P from TOF01 Recon vs TOF12



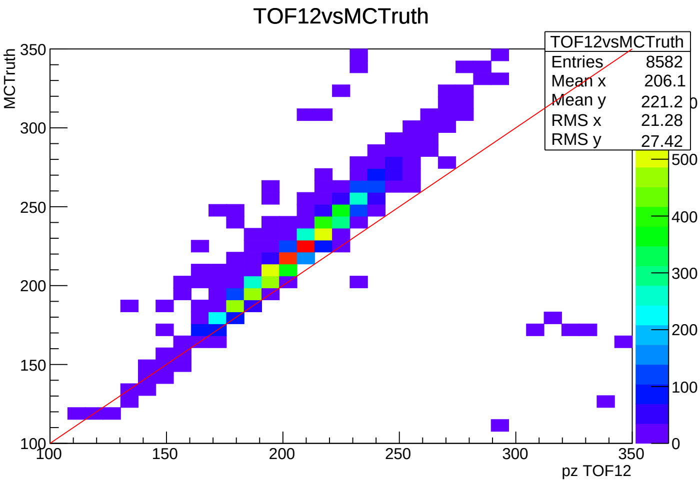
There is a non-trivial amount of energy loss between TOF0 & TOF2

Naive P from TOF01 Recon vs MCTruth at absorber



Virtual plane is at centre of absorber

Naive P from TOF12 Recon against at the absorber



Virtual plane is at centre of absorber

P from TOF01 formula against MCTruth

- Starting from the integral

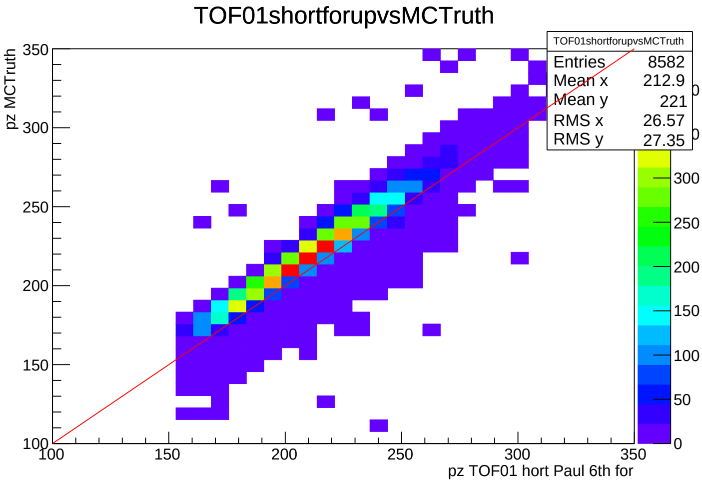
$$\Delta t = \int_{s_1}^{s_2} \frac{ds}{c\beta} \tag{2}$$

- the following expression can be arrived at

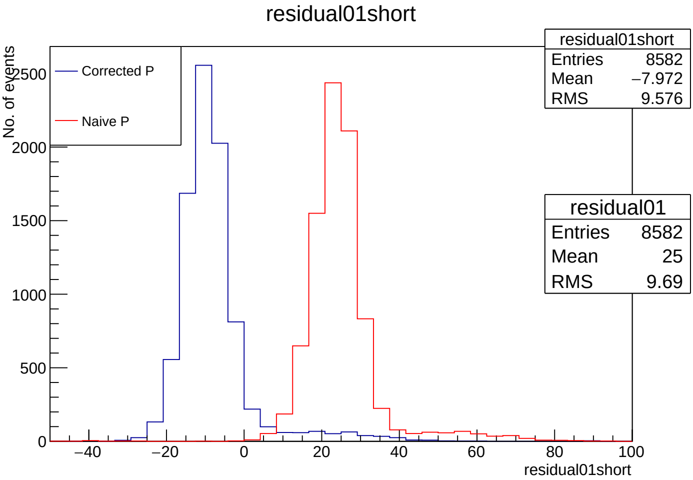
$$\begin{aligned}
a &= x^6 \frac{1}{4m^2c^4} \left(\frac{dE}{dx}\right)^2 (\Delta s_1 + \Delta s_2)^2 \\
b &= -x^3 \frac{c\Delta t}{\Delta s} \frac{1}{mc^2} \frac{dE}{ds} (\Delta s_1 + \Delta s_2) \\
c &= -x^2 \\
d &= \left(\frac{c\Delta t}{\Delta s}\right)^2 - 1 \\
x &= \frac{mc}{p}
\end{aligned} \tag{3}$$

- The full details have been written up in a Note to follow

P from TOF01 formula against MCTruth



P from TOF01 formula against MCTruth

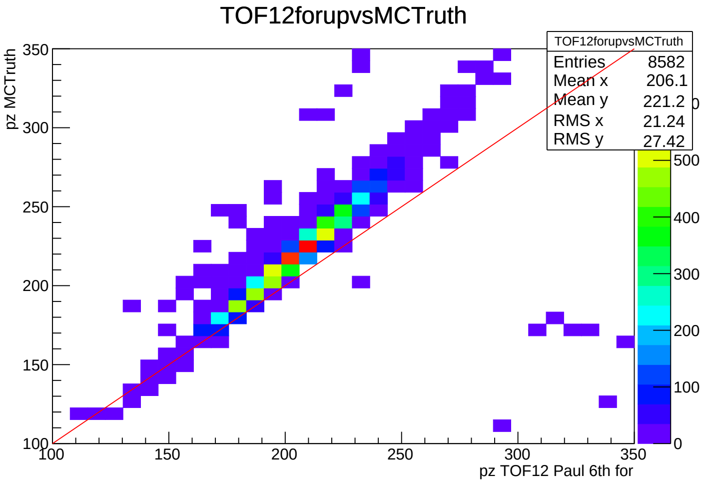


resolution = formula pz - MCTruth pz

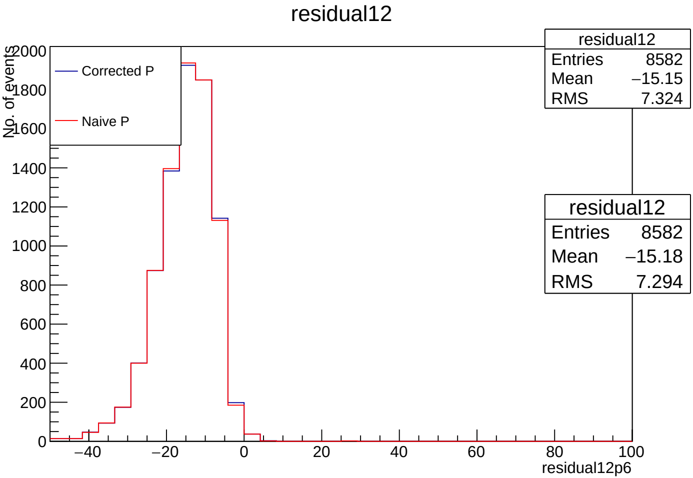
P from TOF12 formula against MCTruth

- This formula can similarly be applied in case where the muon reaches TOF2, the resolution should be better in this case

P from TOF12 formula against MCTruth

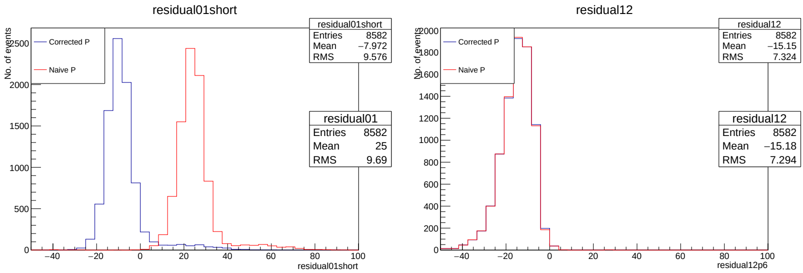


P from TOF12 formula against MCTruth



resolution = formula pz - MCTruth pz

Comparison of resolutions



- As expected the resolution of the measurement between TOF12 is better than that achieved by the TOF01 measurement
- The analysis will proceed by determining the TOF12 measurement *if the muon reaches TOF2*
- If this is not the case the analysis will fall back to TOF01

Conclusions

- Analytic solution for momentum calculation using TOF system has been derived
- Calculation shows improvement over the naive estimate
- Calculated value has been compared to MCTruth value
- Solution for both up and downstream system is available
- Will now implement both in analysis code

dE/dx

- The virtual plane nearest to each of the TOFs is used to measure the average energy of μ at that point
- Energy loss TOF0$\rightarrow$centre of absorber $=$ 35 MeV
- Energy loss TOF1$\rightarrow$TOF2 $=$ 29 MeV
- average $dE/dx = \frac{35}{11.66} = 3.00\text{MeV/m}$
- average $dE/dx = \frac{29}{8.21} = 3.53\text{MeV/m}$

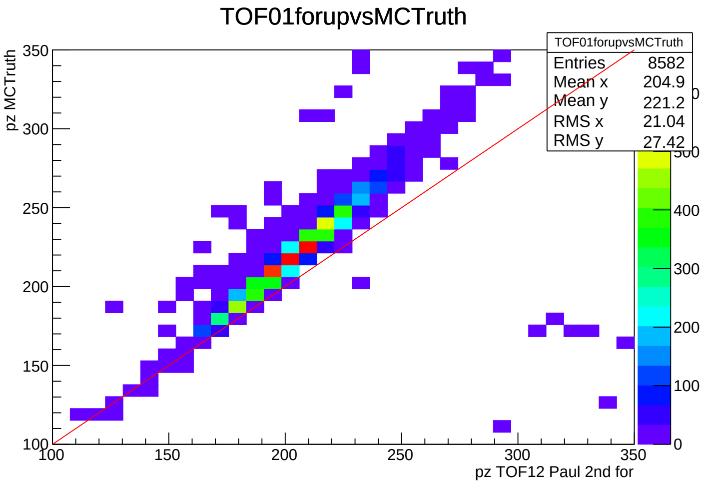
P from 2nd order polynomial TOF12 formula against MCTruth

- If higher order terms in the Taylor expansion are ignored

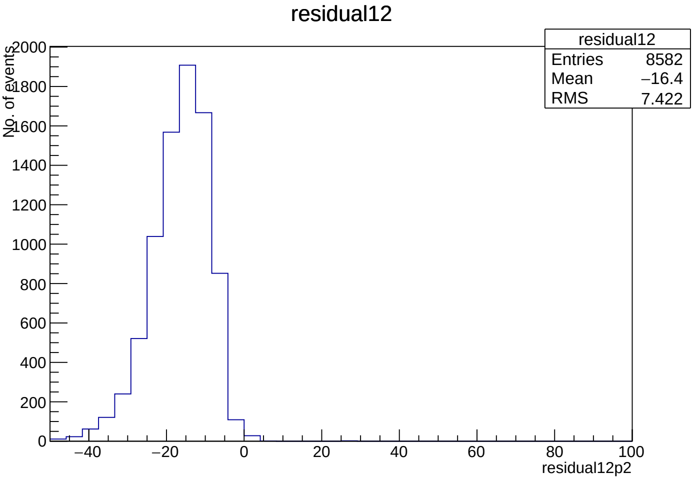
$$p = \frac{mc}{\sqrt{\left(c\frac{\Delta t}{\Delta s}\right)^2 - 1}} \left(1 + \left(\frac{dE}{ds}\right)^2 \frac{(\Delta s_1 + \Delta s_2)^2}{4m^2c^4((\frac{c\Delta t}{\Delta s})^2 - 1)}\right)^{1/2} - \frac{dE}{ds}\frac{c\Delta t}{\Delta sc}\frac{\Delta s_1 + \Delta s_2}{2((\frac{c\Delta t}{\Delta s})^2 - 1)} \tag{4}$$

- However this makes almost no difference to the origianl answer as shown over leaf

P from 2nd order polynomial TOF12 formula against MCTruth



P from 2nd order polynomial TOF12 formula against MCTruth



$$\text{resolution} = \sqrt{(\text{formula pz - MCTruth pz})^2}$$