Pattern Recognition

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August 16, 2017

Abstract

A report on the pattern recognition algorithms used in the Muon Ionization Coolinng Experiment.

1 Introduction

The Muon Ionization Cooling Experiment contains two scintillating fibre trackers which measure beam emittance before and after cooling. Each detector is housed in a superconducting solenoid with field between 2 and 4 T. The trackers are identical, each consists of 5 stations, labelled 1 to 5. Each station consists of three fibre planes at 120° to each other. The longitudinal separation between each station varies, increasing with station number. The upstream tracker, TkUS, is rotated 180° about y with respect to the downstream tracker, TkDS, such that the beam sees station 5 first in TkUS, and station 1 first in TkDS.

2 Geometry

The tracker local coordinate system is a right handed cartesian system, such that z points down the barrel of the tracker, increasing with station number, y points directly upwards, and x points from right to left. x and y have their origin at the centre of the fiducial volume, while z has its origin at the tracker reference surface (the outermost plane of station 1).

The turning angle, ϕ is defined as the angle about the origin in the (x, y), such that it increases as it moves from positive x towards positive y. Hence when looking down the barrel of the tracker from station 1 towards station 5 (in the direction of increasing z) the turning angle increases in the clockwise direction (remembering that x points from right to left to ensure a right handed system).

3 Input

We begin with a set containing either 4 or 5 spacepoints which has passed the previous circle fit selection within pattern recognition. Each spacepoint has an (x, y, z) position associated with it, together with the tracker number (0 for TkUS, 1 for TkDS) and a station number (1 to 5).

4 Helical Transverse Fitting

4.1 Track Model

In the $x - y$ plane the particle track projection is a circle. This may be parameterised in the usual way:

$$\boldsymbol{\theta} = \begin{pmatrix} x_c \\ y_c \\ \rho \end{pmatrix} \tag{1}$$

where (x_c, y_c) are the circle centre coordinates, and ρ is the radius. Let the measurement points be denoted:

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2}$$

The parameters are related to the measure points by:

$$(x - x_c)^2 + (y - y_c)^2 = \rho^2 \tag{3}$$

Note this expression is non-linear in both the coordinates and the parameters.

4.2 Minimisation

Let i denote the i^th measurement, with n total measurements (equal to either 4 or 5). In order to estimate $\boldsymbol{\theta}$ given the $\boldsymbol{x}_i$ the following quantity is minimised:

$$\Delta\chi^2 = \sum_{i=1}^{n} f_i = \sum_{i=1}^{n} \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 - \rho^2} \tag{4}$$

The minimisation rountine employed uses the MINUIT programme, using the MIGRAD algorithm.

4.3 Covariance and Error Propagation

Assuming no correlation between measurement, the measurement covariance matrix is diagonal and defined as:

$$V(\boldsymbol{x}) = \begin{pmatrix} \frac{1}{\sigma_1^2} & \cdots & & \\ \vdots & \frac{1}{\sigma_2^2} & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_n^2} \end{pmatrix} \tag{5}$$

Note that $\sigma_{xy} = \sigma_{yx}$. The σ_i are naively given by the station resolution, or more accurately may be calculated by accounting for multiple scattering. After some work, the multiple scattering errors may be approximated, given in table 1.

The covariance matrix of the parameters can found using the general expression for the propagation of errors:

Station	Error (mm)
1	0.4
2	1.38
3	3.42
4	6.46
5	10.58

Table 1: The approximate station position errors arising from the intrinsic resolution and multiple Coulomb scattering.

$$V(\boldsymbol{\theta}) = J(\boldsymbol{\theta})V(\boldsymbol{x})J^T(\boldsymbol{\theta}) \tag{6}$$

where $V(\boldsymbol{\theta})$ is the covariance matrix of the fitted parameters, and J is the Jacobian matrix. Note: this produces a biased estimate in the non-linear case, due to it being based on the first term only of a Taylor expansion.

The Jacobian matrix is defined as:

$$J(\boldsymbol{\theta}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_c} & \frac{\partial f_1}{\partial y_c} & \frac{\partial f_1}{\partial \rho} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_c} & \frac{\partial f_n}{\partial y_c} & \frac{\partial f_n}{\partial \rho} \end{pmatrix} \tag{7}$$

The components of the Jacobian matrix may be calculated as follows:

$$\frac{\partial f}{\partial x_c} = -\frac{x - x_c}{\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 - \rho^2}} \tag{8}$$

$$\frac{\partial f}{\partial y_c} = -\frac{y - y_c}{\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 - \rho^2}} \tag{9}$$

$$\frac{\partial f}{\partial \rho} = -\frac{\rho}{\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 - \rho^2}} \tag{10}$$

5 Helical Longitudinal Fitting

5.1 Track Model

The full helix may be parameterised using:

$$\boldsymbol{\theta} = \begin{pmatrix} x_c \\ y_c \\ \rho \\ t_s \end{pmatrix} \tag{11}$$

where $t_s = \frac{ds}{dz}$, s being defined as the distance travelled by the track in the $x - y$ plane. Note that the phase advance of the helix (s_0) is not used.

The measurement vector now becomes:

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{12}$$

where z is taken to be the independent variable with no associated error (we assume the detector positions are known to negligible precision). The parameters are then related to the measurements by:

$$x = x_c + (x_0 - x_c)\sin\left(\frac{t_s z}{\rho}\right) - (y_0 - y_c)\cos\left(\frac{t_s z}{\rho}\right) \tag{13}$$

$$y = y_c + (y_0 - y_c)\sin\left(\frac{t_s z}{\rho}\right) + (x_0 - x_c)\cos\left(\frac{t_s z}{\rho}\right) \tag{14}$$

where x_0 and y_0 are the coordinates of the track intersection with the tracker reference station (station 1). This can be set to be the first measurement point, so that $(x_0, y_0) = (x_1, y_1)$.

5.2 Minimisation

Define the predicted value of each x measurment as f_i, and the predicted value of each y measurement as g_i:

$$f_i = x_c + (x_0 - x_c)\sin\left(\frac{t_s z_i}{\rho}\right) - (y_0 - y_c)\cos\left(\frac{t_s z_i}{\rho}\right) \tag{15}$$

$$g_i = y_c + (y_0 - y_c)\sin\left(\frac{t_s z_i}{\rho}\right) + (x_0 - x_c)\cos\left(\frac{t_s z_i}{\rho}\right) \tag{16}$$

Let the actual measurement coordinates be denoted x_i and y_i. The quantity to be minimised is then given by:

$$\Delta\chi^2 = \sum_{i=1}^{n}(f_i - x_i)^2 + (g_i - y_i)^2 \tag{17}$$

The minimisation rountine employed again uses the MINUIT programme, using the MIGRAD algorithm. As the parameters x_c, y_c and ρ were determined at the transverse fit stage, the minimisation need only be performed with respect to t_s.

5.3 Covariance and Error Propagation