TOF Correction

Ryan Bayes, John Nugent

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The time of flight between two points on a curve, s_0 and s_1, is given by

$$\Delta t = \int_{s_0}^{s_1} \frac{ds}{c\beta} \tag{1}$$

where

$$\beta = \frac{pc}{E} = \frac{1}{\sqrt{1 + \frac{m^2c^2}{p^2}}} \tag{2}$$

Suppose that the muon has an energy at some point s_a along the trajectory so that the energy (to first order) is given by :

$$E = E_a - \frac{dE}{ds}(s - s_a). \tag{3}$$

The time of flight between TOF0 and TOF1 can be described by subtracting the time of flight between TOF0 and the centre of the absorber and the time of flight between TOF1 and the centre of the absorber. Then the time of flight between s_0 and s_1 can be expressed as

$$\Delta t = \int_{s_0}^{s_a} \frac{ds}{c\beta} - \int_{s_1}^{s_a} \frac{ds}{c\beta} \tag{4}$$

$$c\Delta t = \int_{s_0}^{s_a} ds \frac{E_a - dE/ds(s - s_a)}{\sqrt{(E_a - dE/ds(s - s_a))^2 - m^2c^4}} - \int_{s_1}^{s_a} ds \frac{E_a - dE/ds(s - s_a)}{\sqrt{(E_a - dE/ds(s - s_a))^2 - m^2c^4}} \tag{5}$$

After integrating by substitution we get:

$$c\Delta t = \frac{1}{dE/ds}(-pc + \sqrt{(E_a - dE/ds(s_0 - s_a))^2 - m^2c^4} + pc - \sqrt{(E_a - dE/ds(s_1 - s_a))^2 - m^2c^4}. \tag{6}$$

Which can be written:

$$\begin{aligned} c\Delta t dE/ds = &\sqrt{(pc)^2 - 2E_a dE/ds(s_0 - s_a) + (dE/ds(s_1 - s_a))^2} \\ &- \sqrt{(pc)^2 - 2E_a dE/ds(s_1 - s_a) + (dE/ds(s_1 - s_a))^2} \end{aligned} \tag{7}$$

Using Taylor expansion this can be expressed as:

$$c\Delta t dE/ds = \frac{E_a}{pc} dE/ds(s_1 - s_0) + \frac{1}{2pc}((dE/ds)^2(s_0^2 - s_1^2 - s_0 s_a + s_1 s_a) \tag{8}$$

Writing in terms of p gives:

$$1+\frac{m^2c^2}{p^2} = \frac{c^2\Delta t^2}{\Delta s^2}-\frac{\Delta t}{\Delta s^2}\frac{1}{p}(dE/ds)(s_0^2-s_1^2-s_0s_a+s_1s_a)+\frac{1}{4(pc)^2\Delta s^2}(dE/ds)^2(s_0^2-s_1^2-s_0s_a+s_1s_a)^2 \tag{9}$$

Multiplying through by p^2 and rearranging gives:

$$0 = p^2(-1+\frac{c^2\Delta t^2}{\Delta s^2})-p\frac{\Delta t}{\Delta s^2}(dE/ds)(s_0^2-s_1^2-s_0s_a+s_1s_a)+\frac{1}{4c^2\Delta s^2}(dE/ds)^2(s_0^2-s_1^2-s_0s_a+s_1s_a)^2-m^2c^2 \tag{10}$$

This polynomial is similar to the one Ryans arrives at in the Note, though naturally given the different starting point and sign differences it is not identical. Finally this can be solved for p with:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \tag{11}$$

where

$$\begin{aligned} a &= (-1+\frac{c^2\Delta t^2}{\Delta s^2}) \\ b &= -\frac{\Delta t}{\Delta s^2}(dE/ds)(s_0^2-s_1^2-s_0s_a+s_1s_a) \\ c &= \frac{1}{4c^2\Delta s^2}(dE/ds)^2(s_0^2-s_1^2-s_0s_a+s_1s_a)^2-m^2c^2 \end{aligned} \tag{12}$$