

# Validating Our Kalman Filter

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# Goals

1. Validate the Statistical properties of the Kalman filter
2. Demonstrate (and therefore understand) the evolution of P-Value distributions
3. Study the behaviour of the track fit in less than ideal conditions

In these slides I mostly look at Smoothed residuals, classic chi-squared residuals, P-Values and Pulls.



# Contents

## 1. “0th-Order” Studies

Done

1D MC State Space, 1D Measurements, 2 Measurement Planes  
Solve the equations analytically and predict residual distributions.

## 2. “1st Order Studies

Finishing

1D MC State Space, 1D Measurements,  $N$  Measurement Planes  
See how process noise, measurement noise, number of measurements, etc affect P-Values

## 3. Straight Track Model

Started

Straight track MC model, assuming Gaussian Process Noise and Measurement Noise  
Aim to recreate P-Values seen in MAUS Data by adding in non-linear processes.

## 4. Helical Track Model

Todo

Helical track MC model assuming Gaussian Process Noise and Measurement Noise  
Aim to recreate P-Values seen in MAUS Data by adding in non-linear processes.



# Analytical Validation

Use a small simple model and try to analytically predict the distributions of the residuals based on the equations for the Kalman Filter.

The comparison between truth and the fitted results should agree with the width of the calculated covariance matrix.

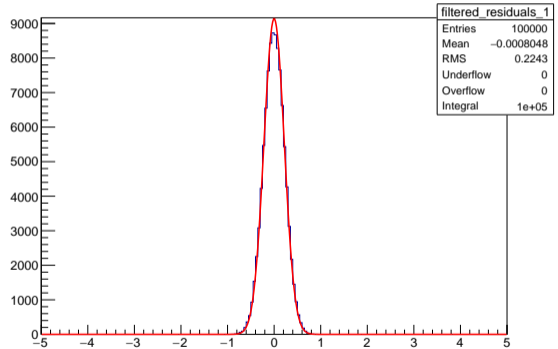
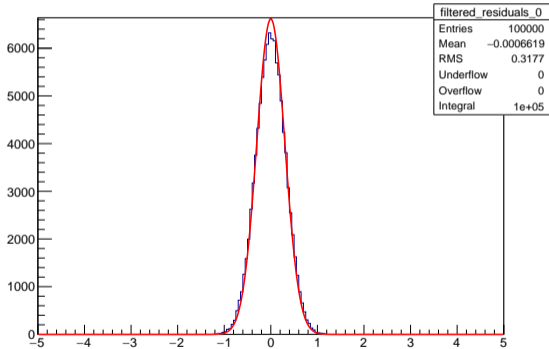
Left to do:

- Add process noise to hand calculations



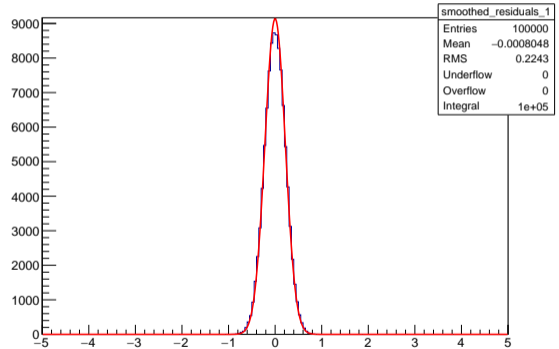
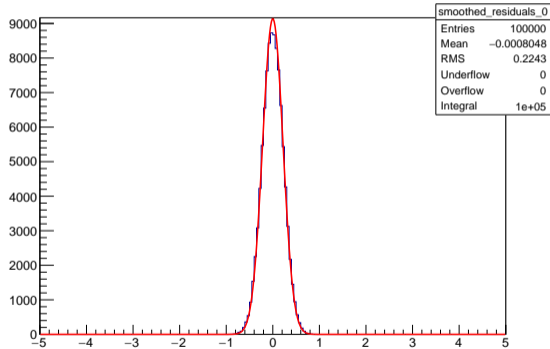
# Analytical Validation

## Filtered Residuals



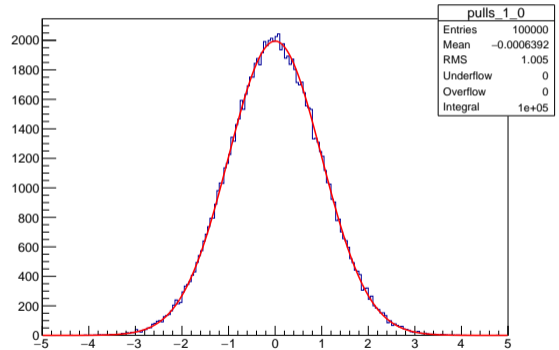
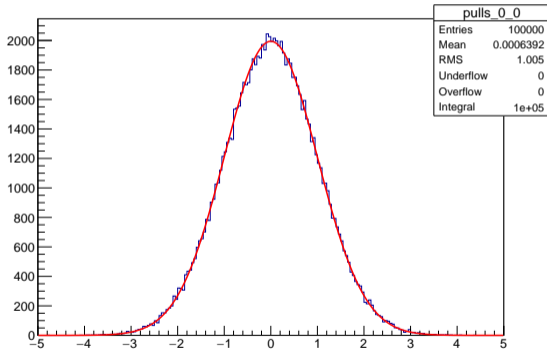
# Analytical Validation

## Smoothed Residuals



# Analytical Validation

## Pulls



# 1D Model

Examine the behaviour of the fit for a 1D system.

Imagine measuring the temperature of a room with a bad thermometer!

Process noise is the random walk of the temperature.

Now try the fit and see what the P-Values, and Chi-Squared values look like.

Left to do:

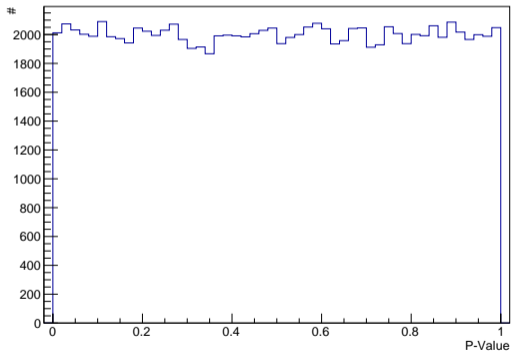
- Non-Uniform P-Values at moderate noise
- Non-Uniform P-Values at low  $N$  - I think this is the seed.



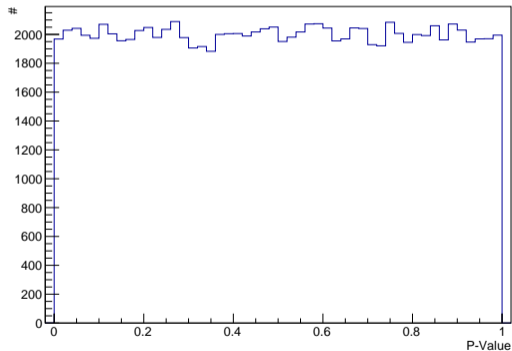


# 1D Model - No Noise

## P-Values (100 Measurements)



Smoothed Residuals

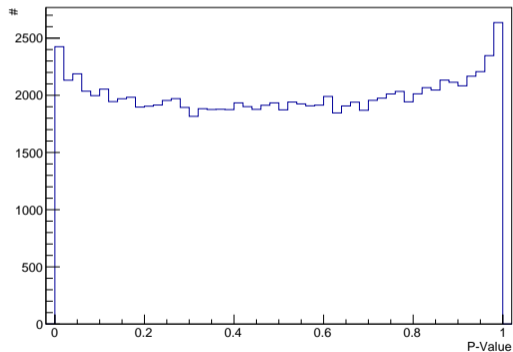


Classical Residuals

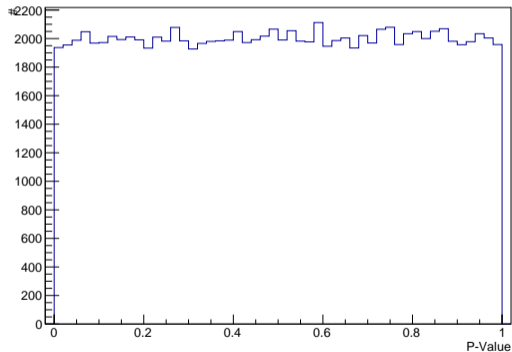


# 1D Model - No Noise

## P-Values (10 Measurements)



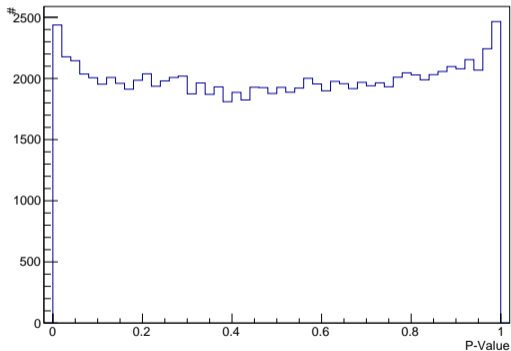
Smoothed Residuals



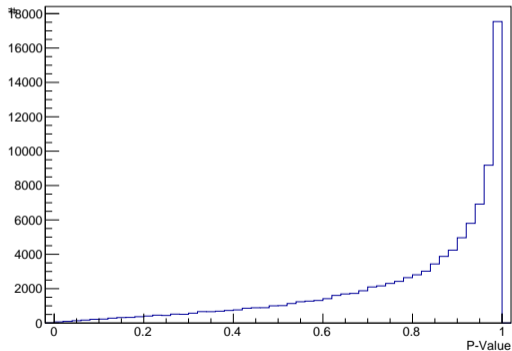
Classical Residuals



# 1D Model - Moderate Noise P-Values (100 Measurements)



Smoothed Residuals



Classical Residuals



## Remaining Questions

- Why do we lose P-Value uniformity for low- $N$ ?
- Why do we lose P-Value uniformity for moderate noise?
- Does this affect our fit results?
- Do the straight and helical track models make sense?

Very nearly at a place to answer these questions.

By the CM the algorithm should be fully validated, available through MAUS-v2.9.1, and well studied.

