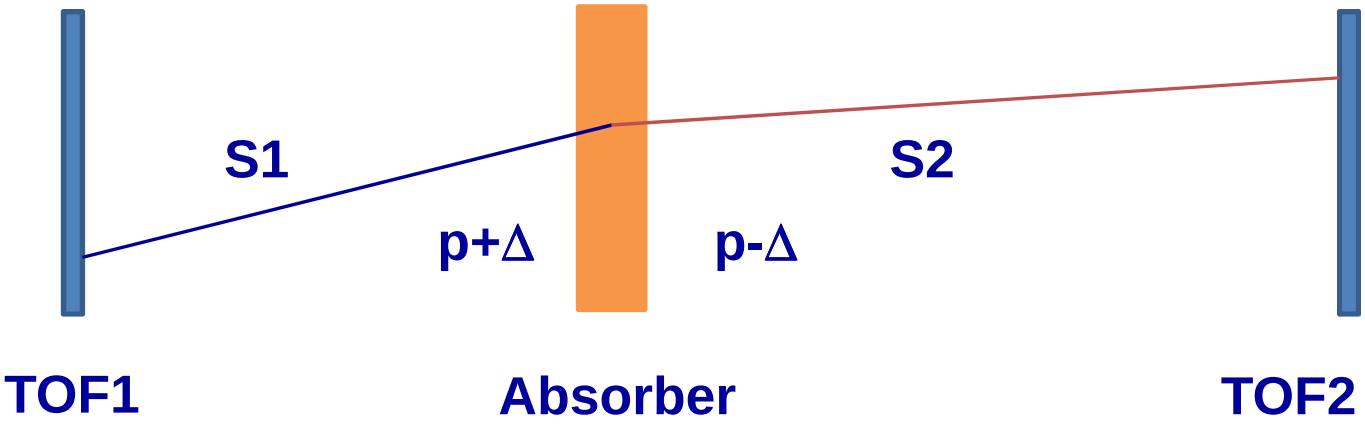
MOMENTUM FROM TOF1 – TOF2 WITH ABSORBER



- S1 and S2 known from geometry (*i.e.* corrected for angle)
- Absorber is thin
- Muon loses momentum 2Δ in absorber
- Δ not known precisely *a priori*
- Can find Time of Flight

TIME OF FLIGHT WITH ABSORBER

$$t_A = s_1 \frac{\sqrt{(p+\Delta)^2 + m^2}}{(p+\Delta)} + s_2 \frac{\sqrt{(p-\Delta)^2 + m^2}}{(p-\Delta)} .$$

Needs estimate of Δ to solve for *p*

Could solve iteratively (perhaps)

Expand in terms of derivatives:

$$
\begin{aligned}
t_A &\approx t_1 + \frac{dt_1}{dp}\Delta + \frac{1}{2}\frac{d^2t_1}{dt^2}\Delta^2 + t_2 - \frac{dt_2}{dp}\Delta + \frac{1}{2}\frac{d^2t_2}{dt^2}\Delta^2 \\
&= t_1 + t_2 + (s_1 - s_2)\frac{d(E/p)}{dp}\Delta + \frac{1}{2}(s_1 + s_2)\frac{d^2(E/p)}{dt^2}\Delta^2 \\
&= t_0 - (s_1 - s_2)\frac{m^2}{p^2E}\Delta + \frac{1}{2}(s_1 + s_2)\frac{m^2}{p^2E}\left(\frac{p}{E} + 2\frac{E}{p}\right)\frac{\Delta^2}{E} \\
&= t_0 + \delta t
\end{aligned}
$$

To first order (only), if *s1* = *s2* then TOF is same as muon that has lost no momentum and Δ enters to second order

➔ Initial estimate of *p* from measured time, *tm*, assuming no energy loss and treat δ*t* as small correction

$$
p_0 = \frac{m(s_1 + s_2)}{\sqrt{t_m^2 - (s_1 + s_2)^2}}. \qquad p_c = p_0 + \frac{dp}{dt}\delta t
$$

Requires an estimate, given *po*, of Δ

Use Bethe-Bloch or lookup table

A bit more algebra to look for cancellations:

$$
\begin{aligned}
p_c &= p_0 + \frac{dp}{dt}\delta t \\
p_c &= p_0 + \left(\frac{dt}{dp}\right)^{-1}\delta t \\
&= p_0 - \frac{1}{s_1+s_2}\frac{Ep^2}{m^2}\delta t \\
&= p_0 + \frac{s_1-s_2}{s_1+s_2}\Delta - \frac{1}{2}\left(\frac{p_0}{E_0} + 2\frac{E_0}{p_0}\right)\frac{\Delta^2}{E_0} \\
&= p_0 + \frac{s_1-s_2}{s_1+s_2}\Delta - \frac{1}{2}\left(\frac{s_1+s_2}{t_m} + 2\frac{t_m}{s_1+s_2}\right)\frac{\Delta^2}{E_0} \\
&= p_0 + \frac{s_1-s_2}{s_1+s_2}\Delta - \left(\frac{s_1+s_2}{t_m} + 2\frac{t_m}{s_1+s_2}\right)\frac{s_1+s_2}{2p_0t_m}\Delta^2 .
\end{aligned}
$$

Does it work?

Some test cases:

S1 = 4m, S2 = 5m, 2x dE/dX

S1 = 5m, S2 = 4m, 2x dE/dX

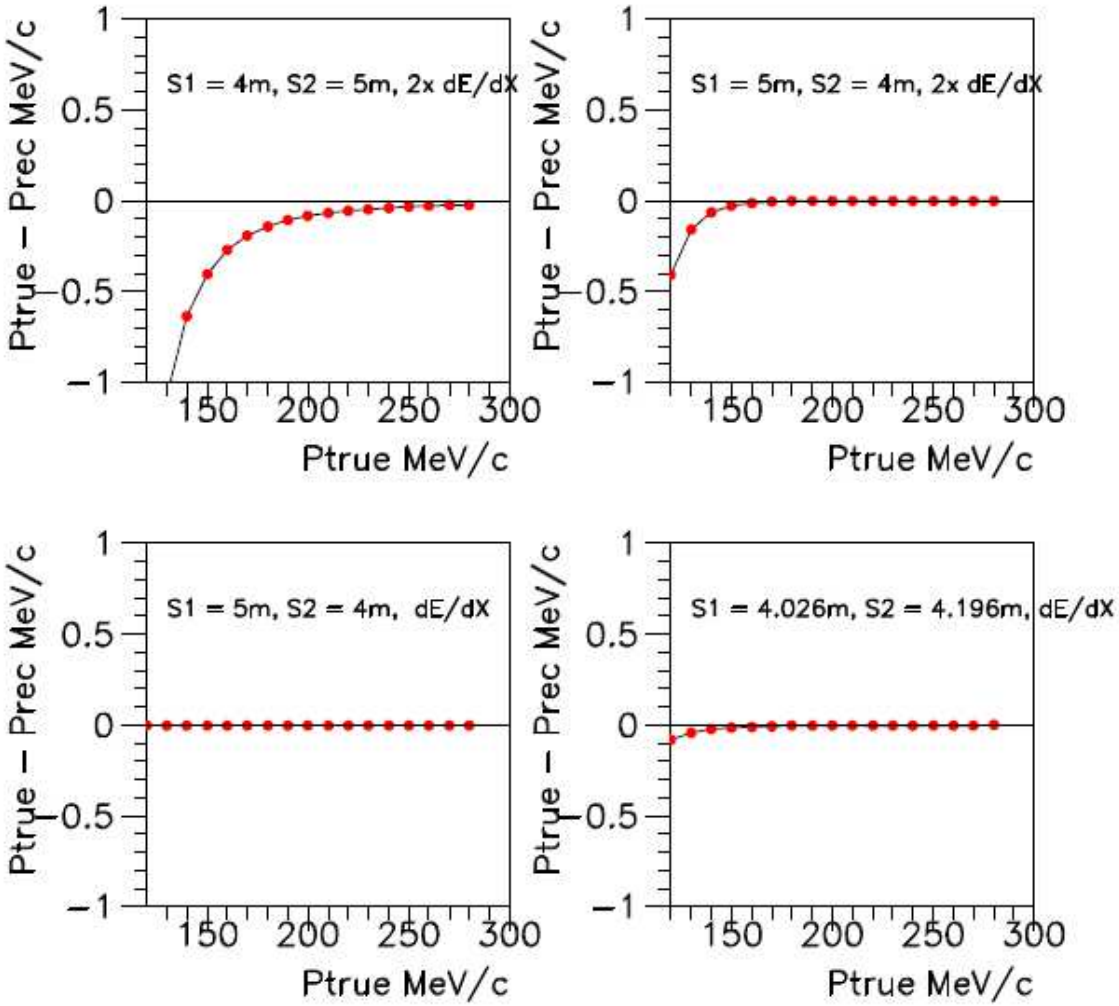
S1 = 5m, S2 = 4m, 1x dE/dX

S1 = 4.026m, S2 = 4.196m, 1x dE/dX = Nominal on axis

***Cheated a bit* with dE/dX – eyeball 1/beta^2 fit to PDG values for LiH x (8/7) for Li(6)H**

No stochastics – simply mean values

Stochastics will determine resolution



Looks good to < 1 MeV/*c* …..

?? Iterate once do better ??

- **Procedure seems to work**
  - **At least on average**
- **Preferable to calibrating TOF – momentum with MC**
  - **Could / should be checked against MC**
- **Requires**
  - **Muon path lengths to be known**
  - **TOFs properly calibrated, electron peak known etc.**
- **Would probably work with (not quite so thin) LH2 absorber**
  - **Possibly extra small correction??**
    - **More algebra…**