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(Root) Mean Square Angles of Scattering

Formal mean square space (3D) angle is

$$< \theta^2 >= \int_{\theta_{min}}^{\theta_{max}} \theta^2 P(\theta) d\Omega$$

Muons scatter from (screened) nucleus and the atomic electrons.

- θ_{min} defined by 'size' of atom $\sim 10^{-5}$ radians
- θ_{max} roughly defined by size of nucleus for μ–N scattering
- $\theta_{max} \sim 1$ (radian) for low Z, $p_\mu = 200$ MeV/c
- θ_{max} determined by *kinematics* for μ – e scattering
- $\theta^e_{max} \sim 4$ milliradians for μ – e scattering
- For a given atomic species N and e scattering probabilities in ratio Z^2:Z
- *We don't accept muons up to (anything like)* θ_{max} *– only a few x 10 mr at best*

Rossi et al. and the PDG expression

The scattering probability, $P(\theta)$, is directly proportional to the single scattering cross-section, $\sigma(\theta)$.

Rossi wanted a simple and convenient – but approximate – expression. He assumed:

- The Rutherford cross-section: $d\sigma \propto Z^2 \frac{d\Omega}{\theta^4}$
- A hard cutoff at θ_{min}
- Ignored a factor $(\frac{Z}{A})^{\frac{1}{6}}$ in a log term
- Noted that the result could be expressed in terms of radiation length, X_0
- Fudged the electron contribution (and ignored kinematic limit) by saying:
  - Increase the scattering probability by $Z^2 \rightarrow Z(Z+1)$
  - Use (as is done) $Z(Z+1)$ in place of Z^2 in X_0
- Obtained the original expression with a coefficient of 21 (15 for projected angle)

- Z/A increases from (1 in Hydrogen only) 2 to 2.5 across periodic table
- $Z(Z+1)/Z$ decreases from 2 to very small across periodic table
- Rossi expression better at high Z

- Highland, following Rossi, observed that there is a Z dependence (but ignored electron kinematics) at low Z and suggested a better coefficient is 13.6 (in 2D) *for all Z*
- Obtained the log term in the current PDG expression
- PDG expression modified - but no explicit Z dependence.
- No Z dependence
- *Caveats*
  - The 'small print' says the PDG expression applies to 98% (check) of the distribution
  - The PDG expression does not 'convolute':
  - $< \theta^2(2t) > \neq 2 < \theta^2(t) >$

- (This is all in Tim Carlisle's thesis)
- Following Rossi, but more carefully, there is a strong Z dependence of $< \theta^2 >$ at low Z
- (Analytic integral, not MC)
- For LH2 the scattering is only 70 - 80% of PDG prediction
- Even when
  - The correct electron scattering kinematic limit is included
  - Better ('Wentzel') cross-section than Rutherford is used
  - Cross-section used is $\sim$ same cross-section as Molière
- ELMS confirms this for LH2
- Molière does not give an expression for $< \theta^2 >$
- but gives a calculable width of the Gaussian part of the distribution

I am very surprised that we seem to be measuring widths for LiH that are greater than any models predict. In general MUSCATT found the low-Z distributions narrower than expected.