Magnetic Alignment

Yingpeng Song

Jan 26, 2017

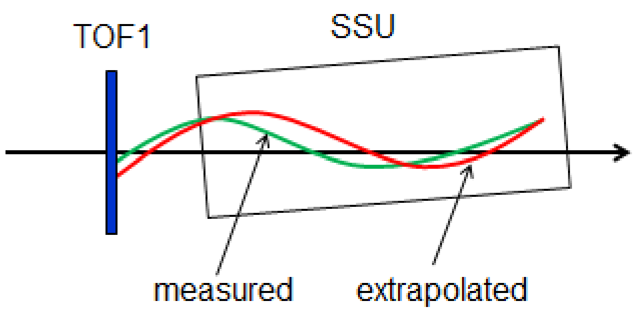
SSU misalignment

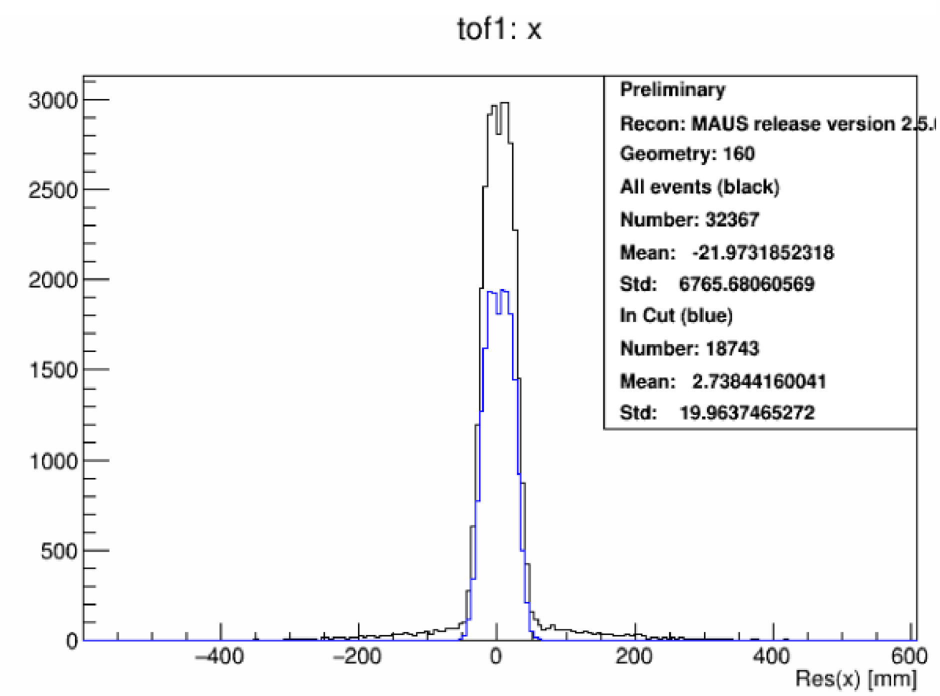
► Beam center residual at TOF1 (run 7469)

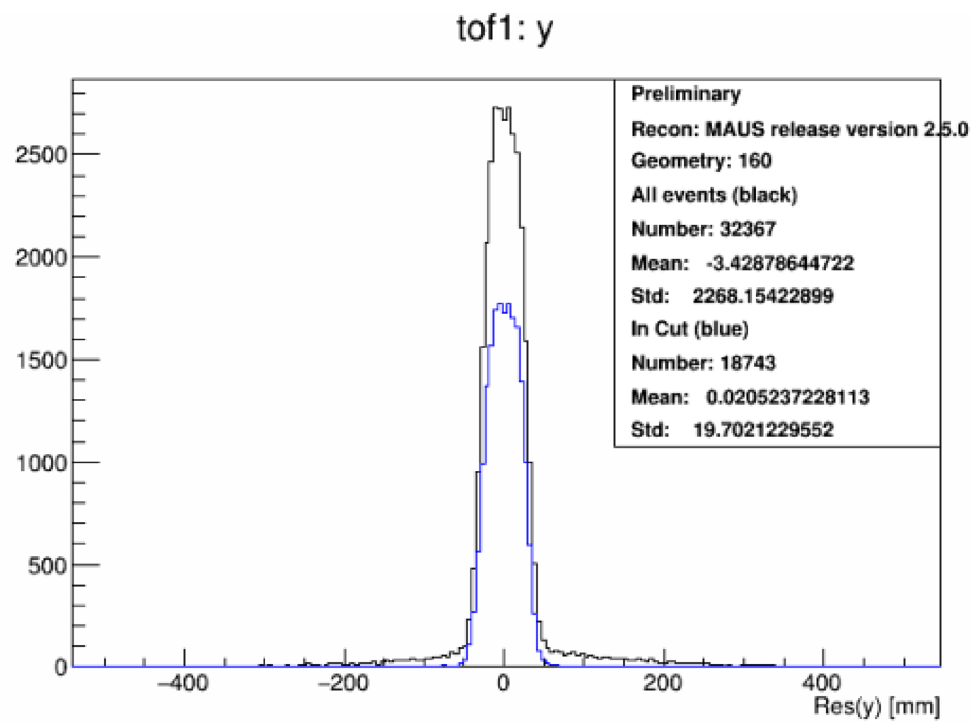
Residual = (measured – extrapolated)

Measured (data): real field map in the hall

Extrapolated (MC): survey of the field map







Misalignment calculation

► Particle propagation in misaligned element

Propagation

① $(x_{in}, x'_{in}, y_{in}, y'_{in})$

↓ Coordinate rotation

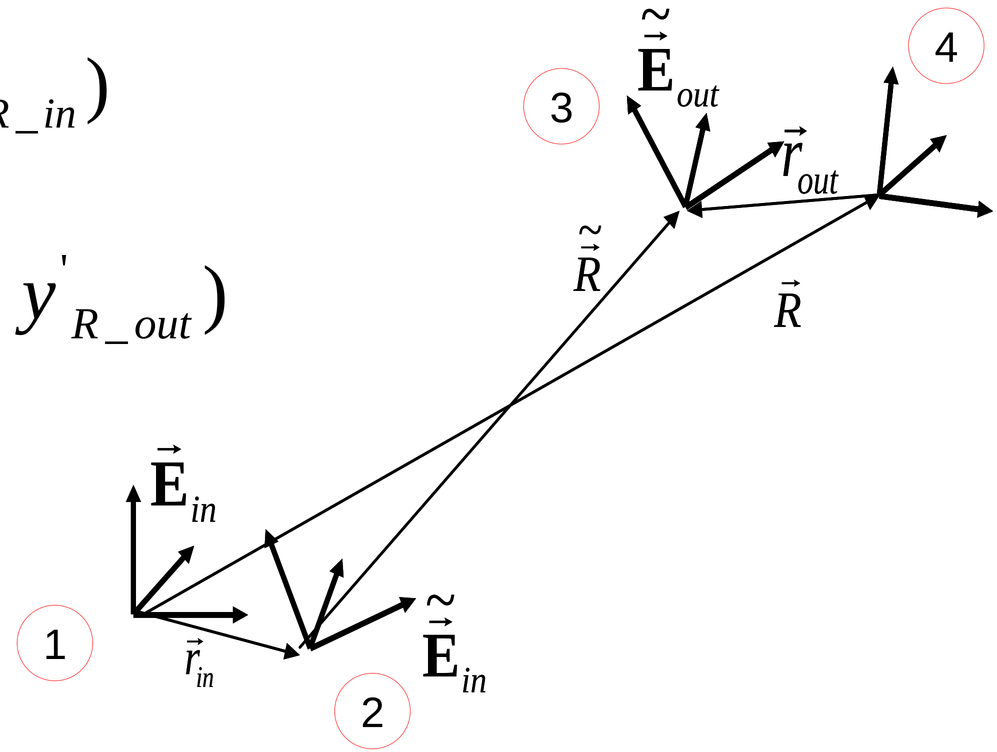
② $(x_{R\_in}, x'_{R\_in}, y_{R\_in}, y'_{R\_in})$

↓ Matrix optics

③ $(x_{R\_out}, x'_{R\_out}, y_{R\_out}, y'_{R\_out})$

↓ Coordinate rotation

④ $(x_{out}, x'_{out}, y_{out}, y'_{out})$



Misalignment calculation

► solenoid

$$\Delta u_1 = \begin{pmatrix} (1-m_{11})\Delta x - m_{13}\Delta y + \left(m_{14} - \frac{m_{13}}{2}L\right)\phi_x + \left(\frac{1+m_{11}}{2}L - m_{12}\right)\phi_y \\ -m_{31}\Delta x + (1-m_{33})\Delta y + \left(m_{34} - \frac{1+m_{33}}{2}L\right)\phi_x + \left(\frac{m_{31}}{2}L - m_{32}\right)\phi_y \\ 0 \end{pmatrix}$$

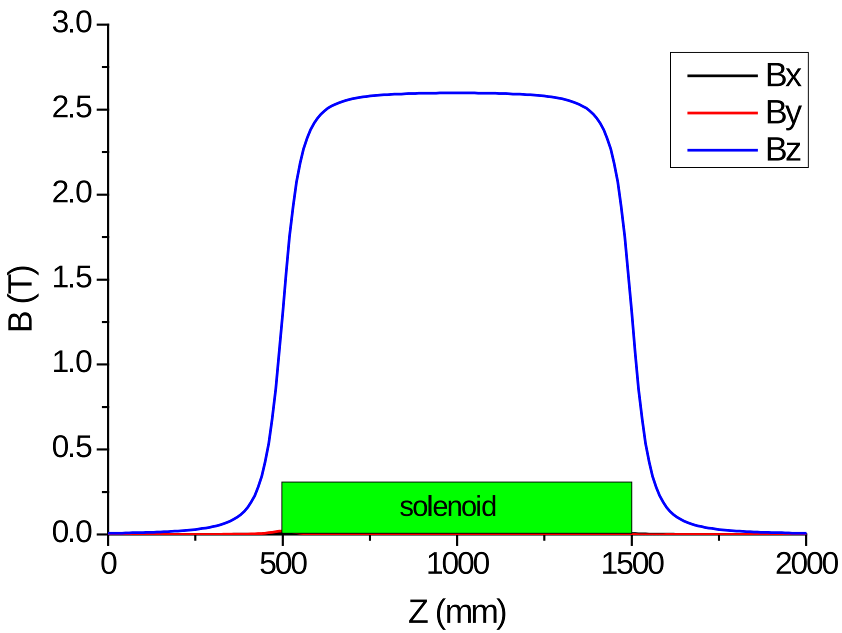
$$\Delta p_1 = \begin{pmatrix} -m_{21}\Delta x - m_{23}\Delta y + \left(-\frac{m_{23}L}{2} + m_{24}\right)\phi_x + \left(\frac{m_{21}L}{2} - m_{22} + 1\right)\phi_y \\ -m_{41}\Delta x - m_{43}\Delta y + \left(-\frac{m_{43}L}{2} + m_{44} - 1\right)\phi_x + \left(\frac{m_{41}L}{2} - m_{42}\right)\phi_y \\ 0 \end{pmatrix}$$

From Cai Meng , Error analysis and beam loss mechanisms of C-ADS linac , PhD thesis

Crosscheck

► Formula check

1. Test by a single solenoid
2. Hard-edge mode used in numerical calculation



	G4Beamline	TURTLE	Numerical
X+2.5mm	2.306/-0.730	2.337/-0.718	2.292/-0.691
Y+2.5mm	0.725/2.307	0.737/2.264	0.691/2.292
X’+2.5mrad	1.348/-2.266	------	1.575/-1.725
Y’+2.5mrad	2.261/1.349	------	1.725/1.575

Hard-edge mode

- Simulation method
  - Single particle propagation in a single solenoid, with the same input vector, then calculate output vector of the particle by two methods, matrix method and realistic field map method(G4BL).
  - Discrepancy value = matrix method – G4BL simulation

Matrix definition used in simulation:

First-Order Solenoid Matrix *

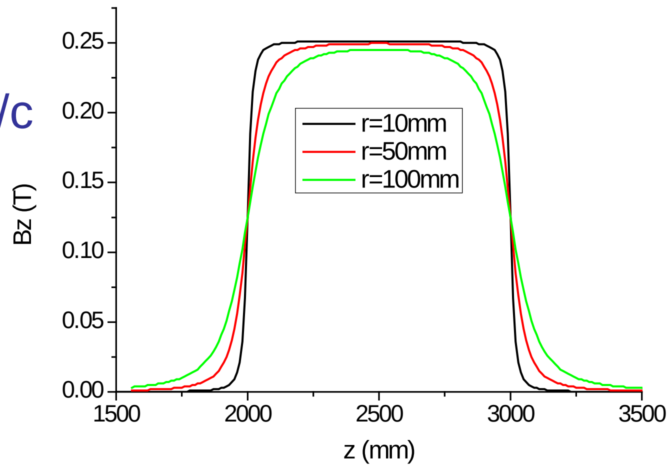
Definitions:
L = effective length of solenoid.
$K = B_0/(2B\rho_0)$, where B_0 is the field inside the solenoid and $(B\rho_0)$ is the magnetic rigidity (momentum) of the central trajectory.
$C = \cos KL$
$S = \sin KL$
γ = relativistic factor

$$R(\text{Solenoid}) = \begin{pmatrix} C^2 & \frac{1}{K}SC & SC & \frac{1}{K}S^2 & 0 & 0 \\ -KSC & C^2 & -KS^2 & SC & 0 & 0 \\ -SC & -\frac{1}{K}S^2 & C^2 & \frac{1}{K}SC & 0 & 0 \\ KS^2 & -SC & -KSC & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Hard-edge mode

▶ Test

- Single particle with momentum 100 MeV/c
- TM1 : Transfer matrix with $\int B \quad dl = C$
- TM2 : Transfer matrix with $\int B^2 \, dl = C$
- FM : Real field map in G4BL

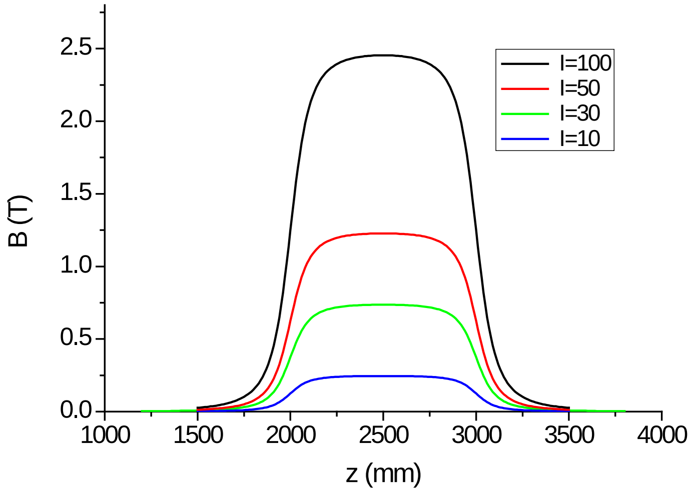


	input	output r_10			output r_50			output r_100		
		TM1	TM2	FM	TM1	TM2	FM	TM1	TM2	FM
x (mm)	3	7.72	7.74	7.65	8.48	8.53	8.34	9.31	9.39	9.10
x' (mrad)	3	3.12	3.10	3.10	3.21	3.13	3.11	3.27	3.16	3.12
y (mm)	3	3.36	3.27	3.45	3.71	3.40	3.74	4.11	3.56	4.12
y' (mrad)	3	1.36	1.31	1.30	1.40	1.25	1.27	1.44	1.20	1.28

Hard-edge mode

► Test

- TM : Transfer matrix with $\int B^2\,dl = C$
- FM : Real field map in G4BL
- L_sol=1m r=100mm



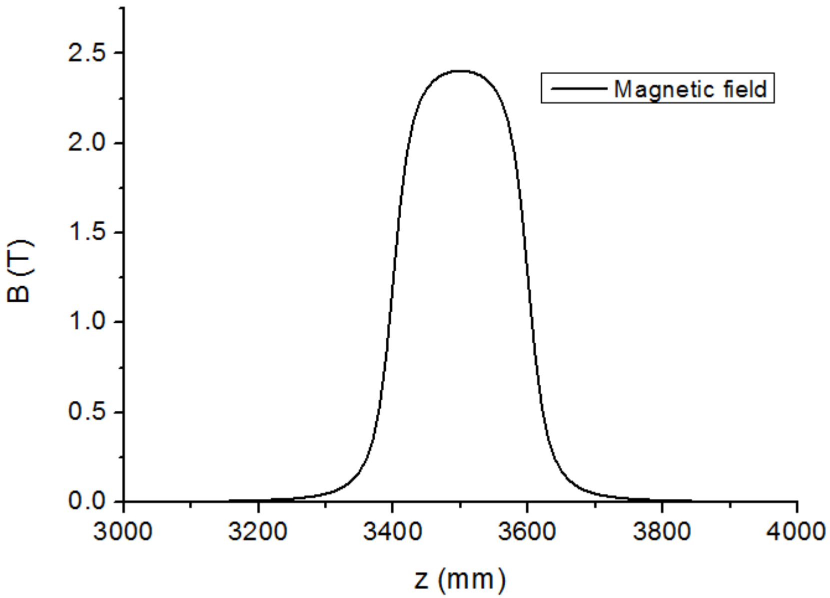
	input	output_10		output_30		output_50		output_100	
		TM	FM	TM	FM	TM	FM	TM	FM
x (mm)	3	9.39	9.10	4.83	4.68	0.08	-2.19	0.58	7.15
x' (mrad)	3	3.16	3.12	-2.05	-3.39	0.56	-5.61	-6.31	1.13
y (mm)	3	3.56	4.12	-3.90	-1.58	1.00	4.12	-0.82	1.53
y' (mrad)	3	1.20	1.28	1.66	1.13	6.56	10.60	8.92	0.05

Hard-edge mode

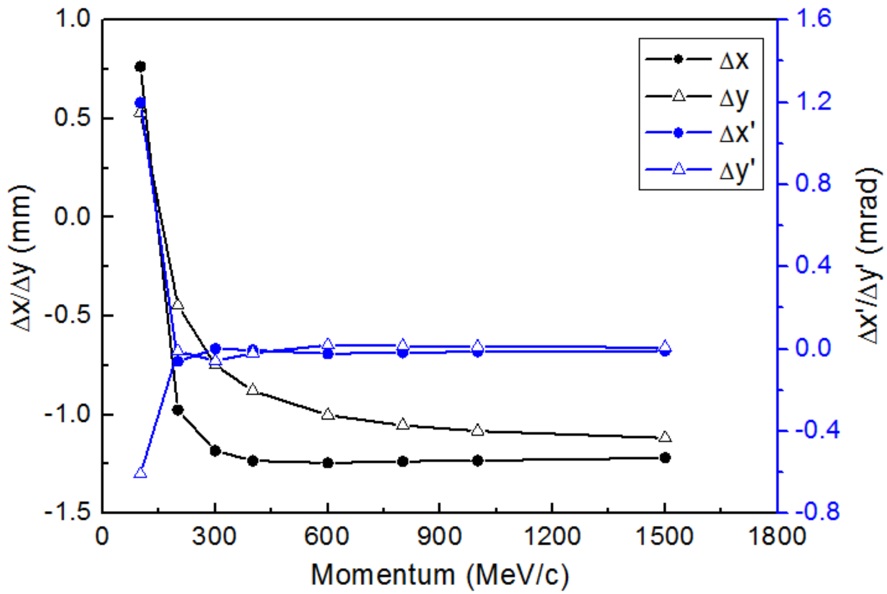
► Discrepancy in short solenoid

Single particle propagation in a single solenoid, with the same input vector, then calculate output vector of the particle

discrepancy value = matrix method – G4BL simulation



A single solenoid with radius 20mm, L=200mm



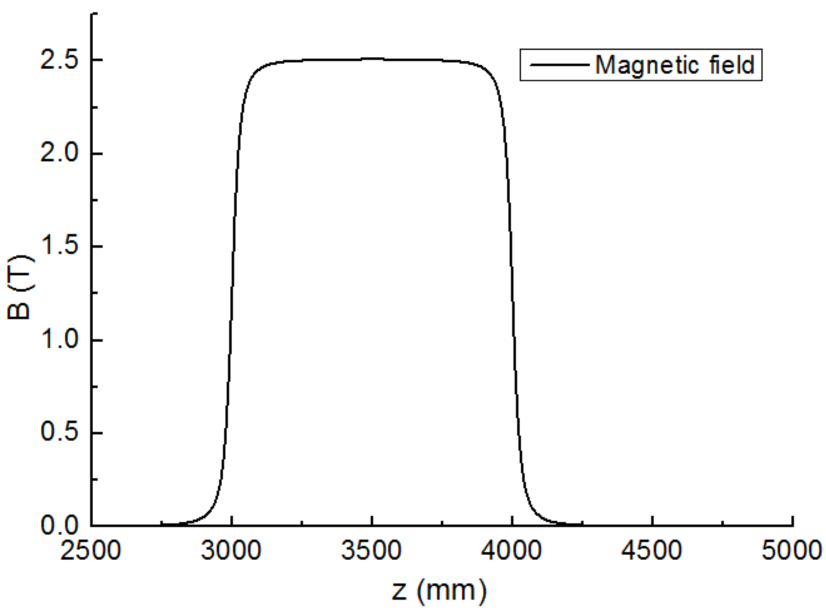
Discrepancy revolution with momentum

Hard-edge mode

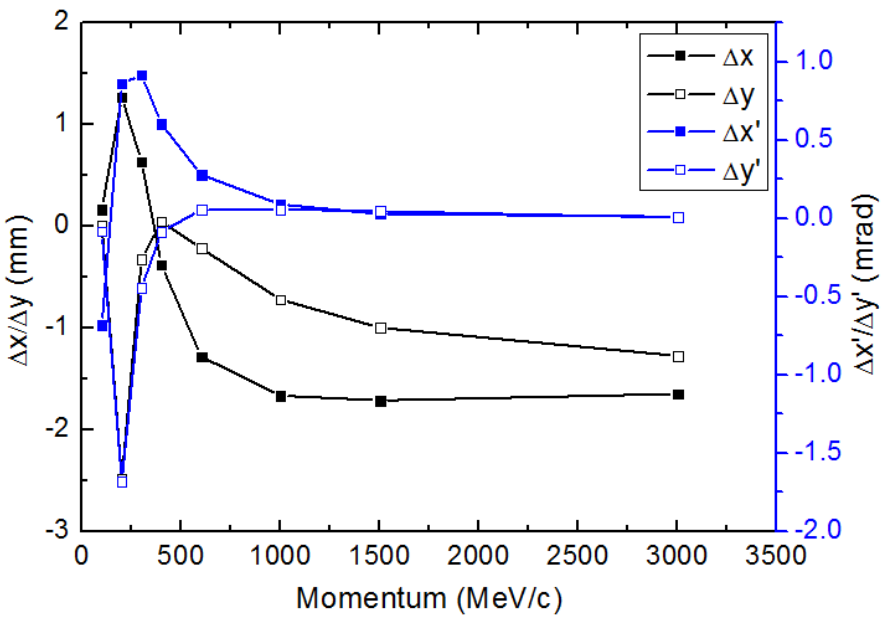
► Discrepancy in long solenoid

Single particle propagation in a single solenoid, with the same input vector, then calculate output vector of the particle

discrepancy value = matrix method – G4BL simulation



A single solenoid with radius 20mm, L=1000mm



Discrepancy revolution with momentum

Hard-edge mode correction

► Soft-edge mode solenoid matrix

$$M_{sol}=\begin{bmatrix} \cos^2\theta & \frac{1}{g}\sin\theta\cos\theta & -\sin\theta\cos\theta & -\frac{2}{g}\sin^2\theta \\ -g\sin\theta\cos\theta & \cos^2\theta & g\sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \frac{1}{g}\sin^2\theta & \cos^2\theta & \frac{1}{g}\sin\theta\cos\theta \\ -g\sin^2\theta & \sin\theta\cos\theta & -g\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} \quad (1)$$

$$L=\frac{1}{B_0}\int_{-\infty}^{\infty} B_z(s)ds \qquad \Phi_{edge} = \left(\frac{q}{2p}\right)^2 \left(\int_{-\infty}^{\infty} B_z^2(s)\,ds - B_0^2 L\right)$$

$$M_{edge} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\Phi_{edge} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\Phi_{edge} & 1 \end{bmatrix} \quad (3)$$

And thus, the transfer matrix of a soft edge solenoid can be expressed as [4]

$$M_{soft\,sol} = M_{edge}\, M_{sol}\, M_{edge} \quad (4)$$

SOLENOID FRINGE FIELD EFFECTS FOR THE NEUTRINO FACTORY LINAC - MAD-X INVESTIGATION, Proceedings of IPAC'10, Kyoto, Japan

Field crosscheck

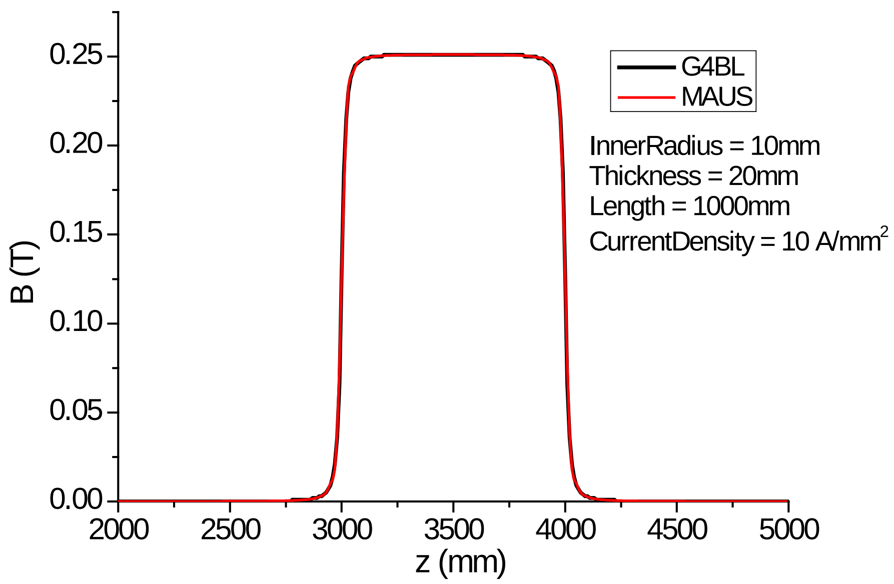
► Magnetic field crosscheck between G4BL and MAUS

A1 = G4BL

0.9670	0.9498	0.1787	0.1755
-0.0336	0.9670	-0.0062	0.1787
-0.1787	-0.1755	0.9670	0.9498
0.0062	-0.1787	-0.0336	0.9670

TM script in MAUS

0.9606	1.1694	0.1913	0.2220
-0.0334	0.9646	-0.0064	0.1703
-0.1914	-0.2220	0.9606	1.1695
0.0064	-0.1703	-0.0335	0.9646

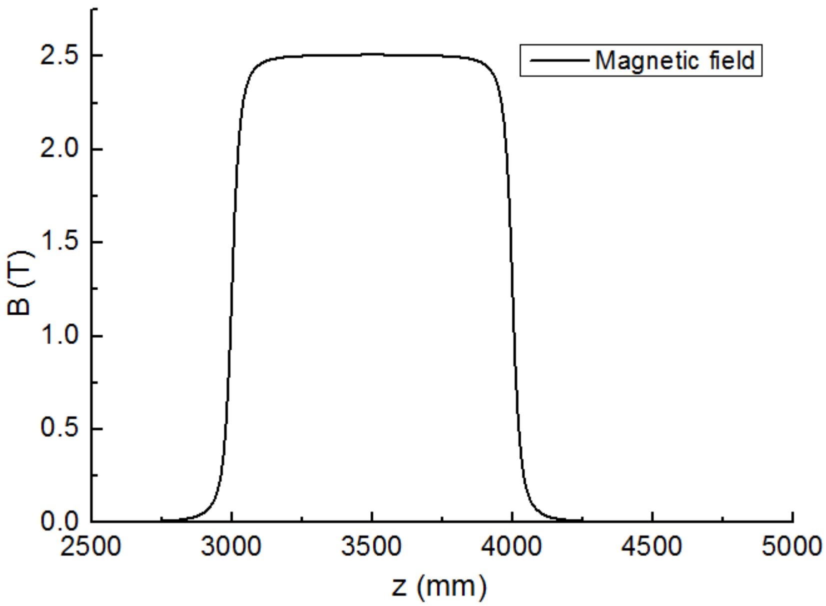


Soft-edge mode Matrix

► Results from different method

Input vector [x,x',y,y'] = [3mm,3mrad,3mm,3mrad]

Propagation from 2750mm to 4250mm




```
Hard-edge mode matrix calculation:

v1 =

    0.6560   -4.6219   -1.0516    7.4096

Soft-edge mode matrix calculation:

v11 =

   -0.8994   -4.9210    1.7185    9.5941

G4BL realistic fieldmap calculation:

v2 =

   -0.8775   -4.9302    1.7322    9.5891
```

Soft-edge mode Matrix

► Offset calculation caused by magnet misalignment

Initial setting: input beam [x,x',y,y'] = [0,0,0,0]

Each time set one misalignment value

Solenoid setting : innerRadius=20mm Length=1000mm
Thickness=20mm currentDensity=100A/mm^2

		x (mm)	y (mm)	x' (mrad)	y' (mrad)	all
Error setting		2	2	2	2	2
formula method	Offset_x	1.563	1.374	1.533	1.985	6.460
	Offset_y	-1.374	1.563	-1.985	1.533	-0.261
G4BL simulation	Offset_x	1.566	1.375	1.533	1.991	6.456
	Offset_y	-1.375	1.566	-1.991	1.533	-0.275

Soft-edge mode Matrix

► Offset calculation caused by magnet misalignment

Initial setting: input beam [x,x',y,y'] = [0,0,0,0]

Each time set one misalignment value

Solenoid setting : innerRadius=100mm Length=1000mm Thickness=20mm currentDensity=100A/mm^2

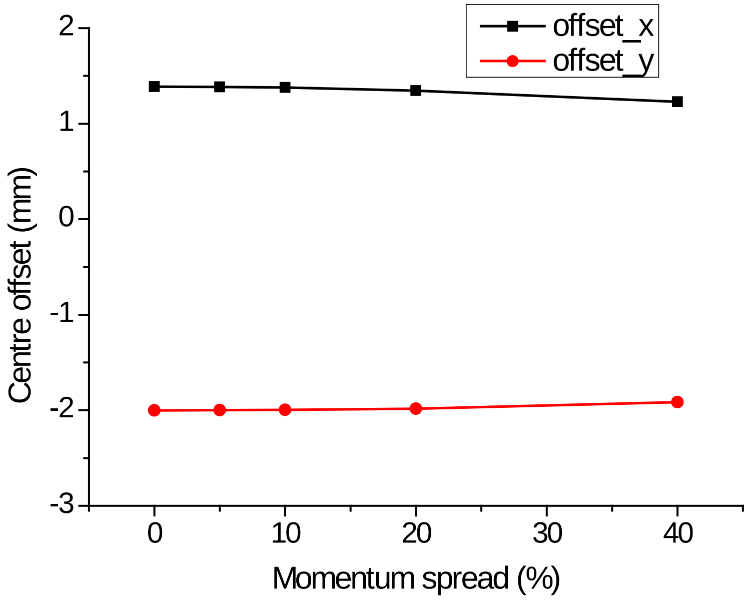
		x (mm)	y (mm)	x' (mrad)	y' (mrad)	all
Error setting		2	2	2	2	2
formula method	Offset_x	1.388	2.001	1.900	2.581	7.871
	Offset_y	-2.001	1.388	-2.581	1.900	-1.295
G4BL simulation	Offset_x	1.388	2.001	1.931	2.623	7.837
	Offset_y	-2.001	1.388	-2.623	1.931	-1.291

Soft-edge mode Matrix

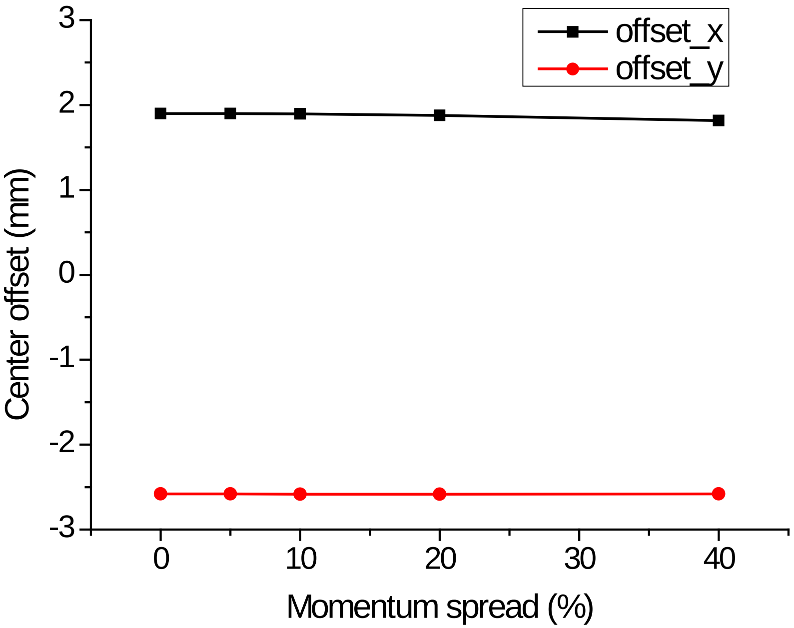
► Momentum spread sensitivity

Incident beam info: 1000 pion, uniform distribution

With RMS emittance 1E-7πmm mrad



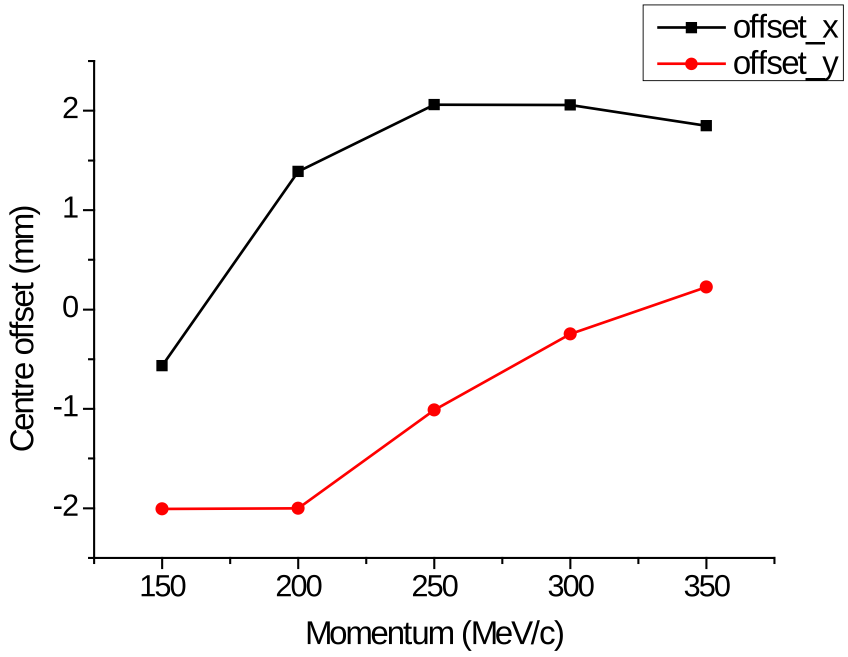
With 2mm translation on x



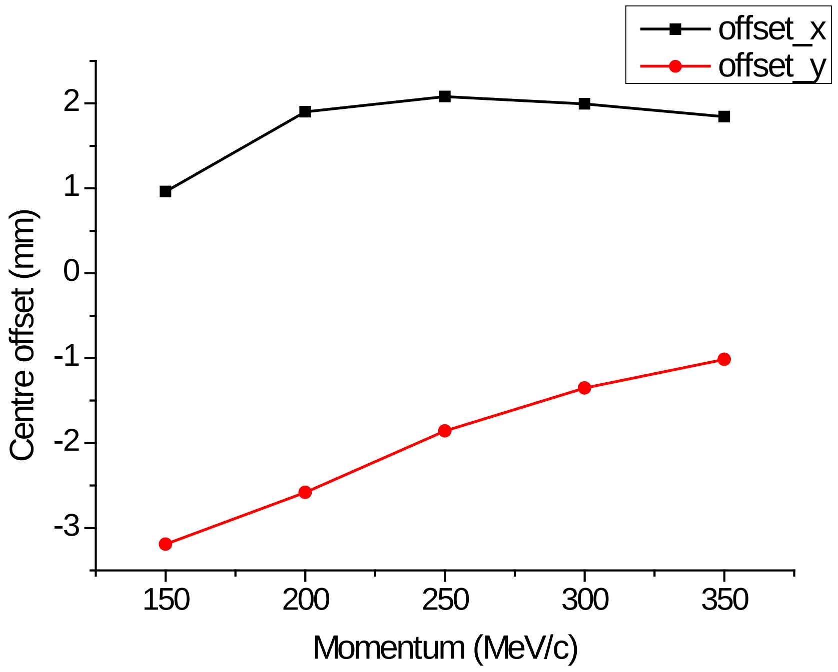
With 2mrad rotation wrt x axis

Soft-edge mode Matrix

► Momentum dependent



With 2mm translation on x



With 2mrad rotation wrt x axis

► Conclusions

- Hard-edge mode approximation has a good performance when the fringe field extends not too much and small KL value(suggestion<0.3 from Theory and design of charged particle beams, Martin Reiser), $K = B_0/2B\rho_0$, L is the effective length of the solenoid.
- When the fringe field extends seriously, we need to do correction on positions of output, the divergence will not be affected.
- Soft-edge mode matrix can make formula work well, also L in formula is realistic length of the field length rather than effective length.