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# Projected angle of scattering

The (2D) projected angle of scattering is the angle between the incident (upstream) and the scattered track vectors projected onto a plane *containing the incident track*. We have used the angles between the track vectors in the $x - z$ and $y - z$ planes of the experimental coordinate system. These are only the true projected angles if the incident muon has no component of momentum in a direction perpendicular to these planes, i.e. the $y$ and $x$ directions respectively.

To obtain the correct projected angle, a 'plane of projection' must be defined for each incoming muon. There is an infinite number of such planes, so there is some arbitrariness.

Let $\vec{u} = (u_x, u_y, u_z)$ be the normalised vector representing the upstream muon direction. The components of $\vec{u}$ are simply its direction cosines, i.e. $u_x = p_x/|p|$ etc..

Define $\vec{d}$ similarly to describe the downstream muon.

To construct a plane of projection which contains the incoming muon, we can choose *any* arbitrary vector, $\vec{s}$ say, which is non-colinear with $\vec{u}$. The vector product $\vec{v} = \vec{s} \times \vec{u}$ is perpendicular to $\vec{u}$ and the plane is defined by $\vec{u}$ and $\vec{v}$. The new direction vector $\vec{v}$ should be normalised: $\vec{v'} = \vec{v}/|v|$.

In the $u - v'$ plane the incident muon is — by construction — moving along the $u$ axis. The tangent of the projected angle in this plane, $\theta_p$, of the downstream, scattered muon is given by the projections of $\vec{d}$ onto the $v'$ and $u$ axes:

$$\tan\theta_p = \frac{\vec{d} \cdot \vec{v'}}{\vec{d} \cdot \vec{u}} \, .$$

How should $s$ be chosen?

We have been using angles measured in the $x-z$ and $y-z$ systems of the experiment (i.e. the MICE coordinate system). Given that the beam muons travel close to the $z$ axis, we can choose $s$ to be along one of the $x$ or $y$ axes. We then recover (almost) the projected angles that we have been using.

Choose $s$ to correspond to looking down the $y$ axis: $\vec{s} = (0, -1, 0)$. Then

$$\vec{v} = \vec{s} \times \vec{u} = (-u_z, 0, u_x)$$

and

$$\vec{v'} = \frac{1}{(u_x^2 + u_z^2)^{\frac{1}{2}}}(-u_z, 0, u_x) \, .$$

The projected angle is given by

$$\begin{aligned}
\tan\theta_p &= \frac{1}{(u_x^2+u_z^2)^{\frac{1}{2}}}\left(\frac{-d_x u_z + d_z u_x}{d_x u_x + d_z u_z + d_y u_y}\right) \\
&= \frac{1}{(u_x^2+u_z^2)^{\frac{1}{2}}}\left(\frac{\frac{u_x}{u_z} - \frac{d_x}{d_z}}{1 + \frac{d_x u_x}{d_z u_z} + \frac{d_y u_y}{d_z u_z}}\right) \\
&\rightarrow \frac{\frac{u_x}{u_z} - \frac{d_x}{d_z}}{1 + \frac{d_x u_x}{d_z u_z}} \\
&\rightarrow \frac{u_x}{u_z} - \frac{d_x}{d_z} \\
\theta_p &\rightarrow \theta_u - \theta_d
\end{aligned}$$

where the first $\rightarrow$ occurs when $u_y$ goes to zero (the upstream track lies in the $x - z$ plane), and the second $\rightarrow$ occurs when the angles in that plane are small. Finally the familiar result is recovered (I should have chosen the opposite sign for $s$).

If $s$ is chosen to be along the $x$ axis, the resulting projected angle would be in a plane close to the $z - y$ plane.

I don't how how large the corrections to the distributions would be with the correct definition of the projected angle, although I would guess that they are small.

I think this is best done (coded) using standard vector manipulation routines, rather than using components explicitly.