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MICE Note 495
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Preliminary dimensions of Focus Coil coils

I have made preliminary fits to the field mapping measurements to find the effective dimensions of the coils in each Focus Coil module.

The fits assume that the two coils in a module are identical block conductors. The field on the axis of a block conductor is given in the appendix. I have allowed for the very slight difference in the numbers of turns in each coil in calculating the current density.

The fitted parameters are:

1. r_1, the inner radius of each coil;
2. $l = z_2 - z_1$, the length of each coil;
3. $t = r_2 - r_1$, the thickness of each coil;
4. s, the separation between the centres of each coil, and
5. f, an overall current scaling factor.

A further trivial fit parameter is z_c, the mid-point between the two coils (in the mapper system).

The fit is to the magnitude of the total field, $B_m = (B_x^2 + B_y^2 + B_z^2)^{\frac{1}{2}}$, measured by the central probe (probe 0) of the mapper. The total field, rather than B_z, is used because the z-axis of the probe is not necessarily parallel to the z axis of the mapper and, in Flip Mode, there is a large B_r midway between the coils.

The total field as a function of the parameters is

$$B_t = (B_z^2(z,r) + B_r^2(z,r))^{\frac{1}{2}}$$

where r is the distance between the probe and the magnetic axis of the module (found previously) and $B_z(z,r)$ and $B_r(z,r)$ are from a first order expansion starting with $B_z(z,0)$; the necessary derivatives are calculated numerically.

Figure 1 shows the fit residuals, $B_m - B_t$, for a fit to the mapper measurements for FC 1 in Flip mode at 150 Amps. The residuals of fits made to FC 1 at 100 Amps, and FC 2 at 100 and 150 Amps have very similar shapes. There are clearly some remaining systematic effects, in particular for $z > 1500\,\text{mm}$ where the residual rises to ~ 50 gauss, which I attribute to the flange for the absorber safety (vacuum) window in the region $z \approx 1547$ to $z \approx 1587\,\mathrm{mm}$[1]. Accordingly the fit was made to the measurements only in the region $800 < z < 1540\,\mathrm{mm}$; it is worse – at least uglier – if the region $z > 1540$ is included.

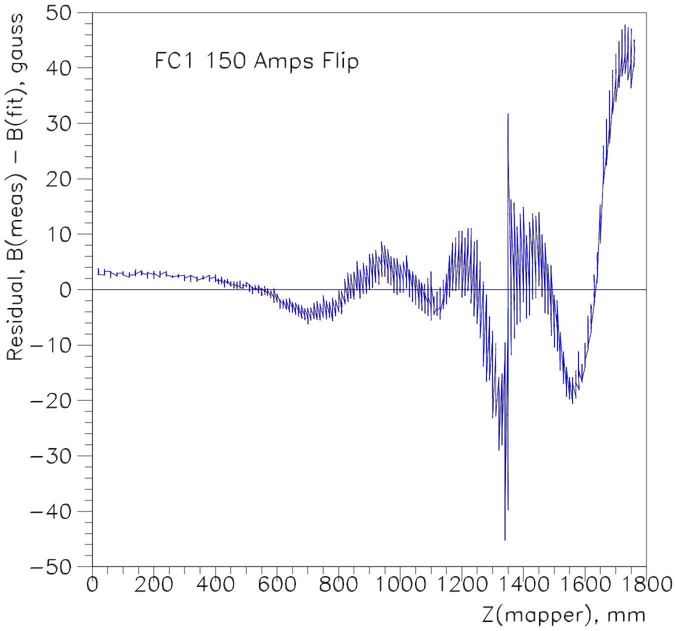

Figure 1: Residuals of fit to FC 1, 150 Amps, Flip mode. The mid-point between the coils, z_c, is at $z = 1348\,\mathrm{mm}$.

Results of fits

The results of the fits, as well as the nominal warm and cold dimensions of the coils are given in Table 1. The current scaling factors are with a few parts in a thousand of unity; there is an apparent current dependence of r_1 and l, although given the systematic effects it is hard to estimate an error on the values. However, I would expect r_1 to increase with current due to magnetic forces, although I don't have an estimate of how big the movement would be.

I would suggest that for practical purposes the average values are used.

[1]The flange is a 316L stainless steel annulus 40 mm thick, 470 mm outer diameter and 320 mm inner diameter. Although SS is nominally 'non-magnetic', it in fact has a μ_r of 1.01 to 1.02. A rough calculation of the field perturbation on the axis of a fully magnetised such ring gives about 60 gauss.

	Module	I	f	r_1	s	l	t	N_{up}	N_{dn}
		Amps		mm	mm	mm	mm	turns	turns
Nominal	warm			267.60	405.40	213.30	94.40		
	cold			266.93	403.78	212.77	94.16		
Fitted	2	150	1.0030	265.86	402.49	212.33	93.73	11257	11256
	2	100	0.9986	265.23	403.77	211.95	93.49		
	1	150	1.0024	265.76	403.41	212.67	93.60	11263	11266
	1	100	0.9985	265.28	404.47	211.69	93.58		
Average			1.00063	265.53	403.54	212.16	93.60		

Table 1: Fitted coil dimensions of FC 1 and FC 2. The last two columns give the numbers of turns in the up- and downstream coils.

Improvements

It may be possible to improve the fits. It is known that there are fewer turns (by up to two in 133) on the five innermost layers of each coil. I tried to include these by subtracting the field due to current loops at the inner edges of each coil but it didn't improve the fits.

The vacuum window flange could, perhaps, be modelled though that might require an FEA calculation, and the μ_r of the steel is not well known. Also there are similar flanges at the upstream end (which were not present when the modules were mapped) and the bore tubes which are fairly substantial pieces of steel.

A complication in the fits is the presence of the logarithmic 'shape' term in the expression for B_z of a block conductor which means that some of the parameters are highly correlated. It may be better to write the fit in terms of ratios, for example r_2/r_1, although this needs some thought.

Appendix: Field on axis of a block coil

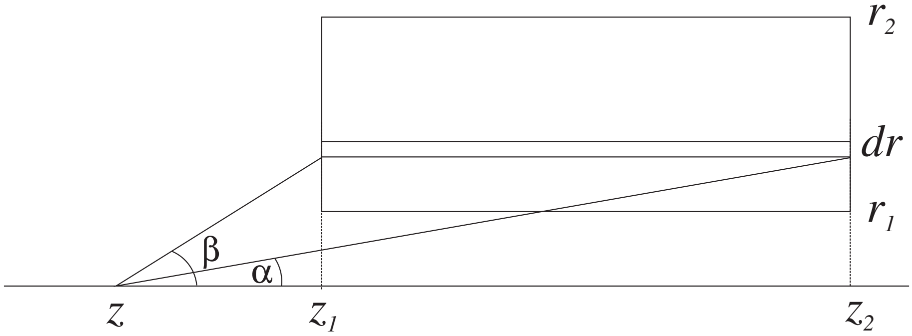

Figure 2: Geometry of the coil.

Consider the block conductor shown in the figure. The coil has N uniformly wound turns and carries a current I. The field at a point z on the axis due to the elementary current sheet between r and $r + dr$ is

$$\begin{aligned} dB_z &= \frac{\mu_0 J}{2}(\cos\alpha - \cos\beta)dr \\ &= \frac{\mu_0 NI}{2(r_2 - r_1)(z_2 - z_1)}(\cos\alpha - \cos\beta)dr \\ &= \frac{\mu_0 NI}{2(r_2 - r_1)(z_2 - z_1)}\left\{\frac{(z_2 - z)}{\sqrt{(z_2 - z)^2 + r^2}} - \frac{(z_1 - z)}{\sqrt{(z_1 - z)^2 + r^2}}\right\} dr\,. \end{aligned}$$

The above expression can be integrated to give

$$\begin{aligned} B_z &= \frac{\mu_0 NI}{2(r_2 - r_1)(z_2 - z_1)}\left\{(z_2 - z)\ln\left[\frac{\sqrt{(z_2 - z)^2 + r_2^2} + r_2}{\sqrt{(z_2 - z)^2 + r_1^2} + r_1}\right]\right. \\ &\qquad \left. -(z_1 - z)\ln\left[\frac{\sqrt{(z_1 - z)^2 + r_2^2} + r_2}{\sqrt{(z_1 - z)^2 + r_1^2} + r_1}\right]\right\}\,. \end{aligned}$$

The terms within the {} represent a ‘shape factor’; the terms outside the {} give the overall scale.

The overall scale depends strongly on the length and thickness of the coil; the shape depends logarithmically on the radii and the length.