1 Error Propagation

The error propagation algorithm extrapolates tracks and associated errors from the tracker to TOF1 and TOF0. Tracks are initially taken as output from the MAUS Kalman-based track reconstruction, with the most upwards point in the reconstructed track considered for analysis. The tracks are compared with measurements in TOF1 and TOF0 to validate the reconstruction.

2 Tracking Algorithm

Integration through the fields is achieved numerically via 4th order Runge-Kutta with a fixed 1 mm step size. The equations of motion are determined using the usual Lorentz force law for magnetic fields and the Bethe-Bloch equation for energy loss. As tracking is backwards, the equations of motion are modified for the appropriate time-parity operation.

While it is not possible to estimate the absolute magnitude of stochastic processes, the uncertainty associated with these processes can be estimated. The track is initially assigned an uncertainty as returned by the Kalman track fitting based on knowledge of the tracker spatial resolution and the quality of the track fit to the detected space points. As the track is integrated through the fields, the resultant envelope is also integrated backwards using the derivatives of the equations of motion to infer the behaviour of the envelope. This technique will be the subject of a future paper. The algorithm has been validated by considering numerical derivatives of the tracking.

As the envelope passes through materials, the width of the envelope is increased by an amount estimated from the Moliere scattering formula. Energy straggling of the energy width of the beam is not included in the analysis.

When the track passes through a detector, the difference between the track and the measured variable, $u_m - u_t$, is calculated and the results are shown in the figures. Additionally, the difference between the track and the measured variable normalised to the estimated errors are plotted, r_n, as given by

$$r = \frac{u_m - u_t}{\sqrt{Var(u_m) + Var(u_t)}} \tag{1}$$

$Var(u_t)$ is the estimated track error and $Var(u_m)$ is the estimated detector resolution.

The tracker has no inherent time resolution. In order to compare time measured in TOF1 with time measured in TOF0, the track is extrapolated back to TOF1 and then the TOF1 time applied. The track time at TOF1 is extrapolated back from TOF1 to TOF0 to make the TOF0 time residual. Additionally, the tracker reconstructed track is compared to the TOF track at TOF1.

Two samples are compared; all events that are successfully extrapolated back to a given region and those tracks that pass the `cut_allPassed` cut and the `cut_tof1_tku_momentum` pid cut.

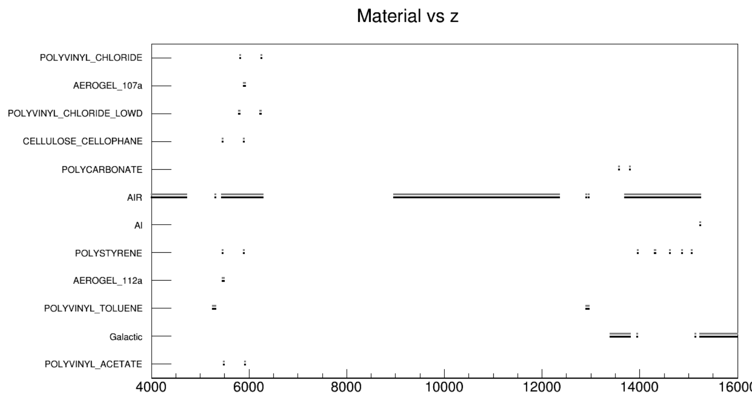


Figure 1: Materials through which MAUS tracks.

3 MAUS Material Model

The on-axis materials through which MAUS tracks are shown in Fig. 1. The list of materials was found by tracking a particle trajectory along the axis of the experiment using Geant4 and recording, at each step, the material through which the particle stepped. Additional materials are modelled for particles passing off the axis of the experiment, for example in the quadrupole aperture. However, only those named materials that are present in this Fig. 1 are included in the error propagation. This prevents large errors arising from tracks that graze apertures but would otherwise be included in the beamline.

4 TOF1 Residuals

The TOF1 position residuals are shown in Fig. 2 and 3. As timing information is taken from TOF1, no time residual is shown at TOF1.

The mean of the position residual is significantly biased in both x and y. The origin of the bias remains to be studied. Some thoughts are outlined below. *ROGERS: tackle the bias or treat as systematic?*

- A bias could be introduced due to asymmetric scattering, induced by one side of the beam scraping more than the other side of the beam
- A bias could be introduced due to misalignment of the detectors. It is noted that the detectors have been aligned by analysis of straight tracks data.
- A bias could be introduced due to misalignment of the solenoid with respect to the beamline. This may introduce a bias in the extrapolation of tracks from the tracker to TOF1.

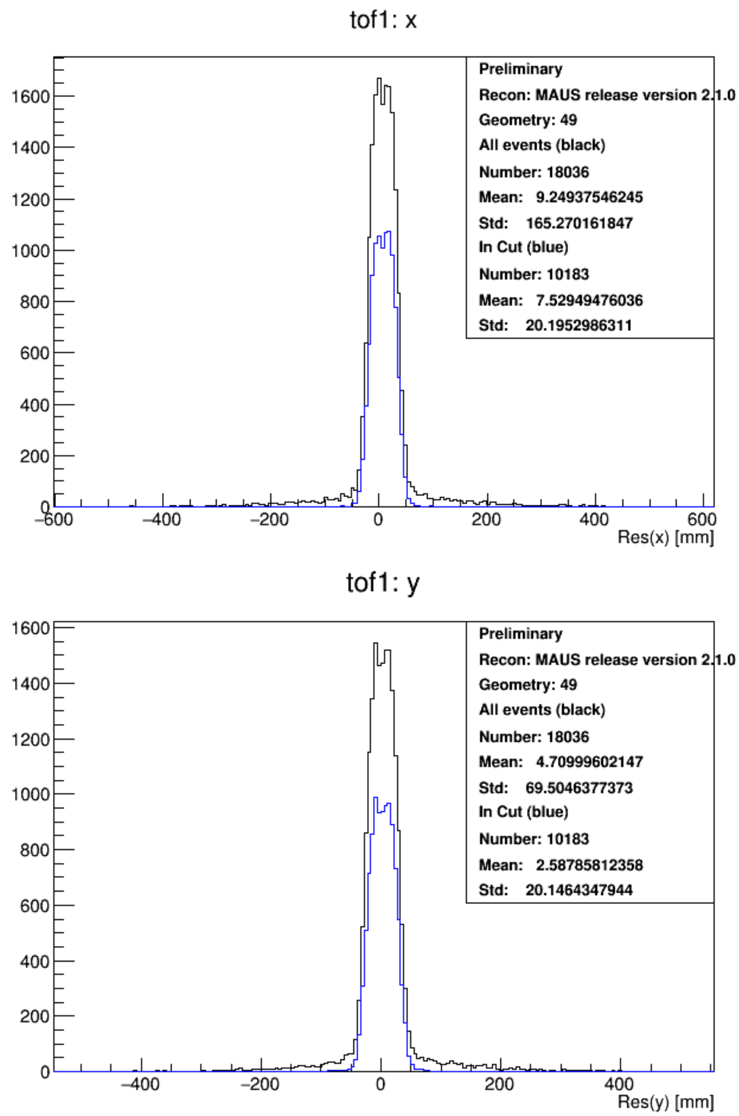


Figure 2: TOF1 residuals.

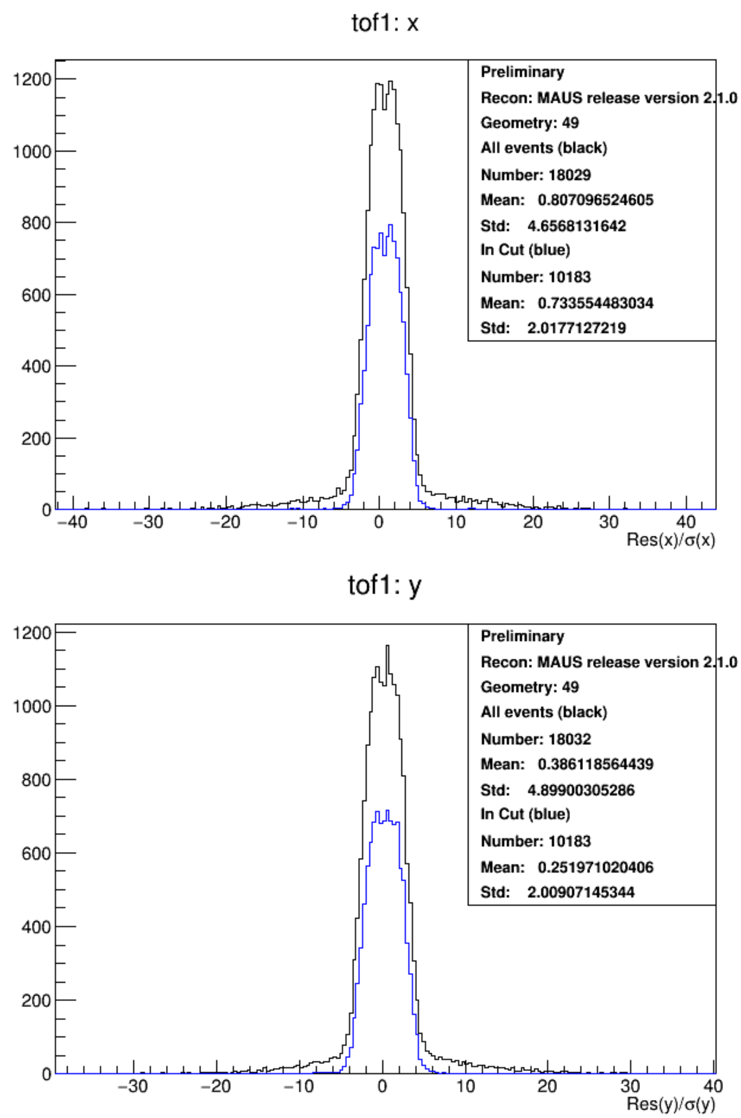


Figure 3: TOF1 normalised residuals.

- A bias could be introduced due to misalignment of the solenoid with respect to the tracker. This may introduce a bias in the tracker reconstruction.

The width of the position residual is twice as wide as would be expected due to the known errors. The source of this width remains to be studied. Again, possible sources are listed:

- The error on the initial reconstructed track could be an underestimate.
- Additional, unaccounted material could induce some scattering.

5 TOF0 Residuals

The TOF0 residuals are interesting. The position residual at TOF0 shows an asymmetric double peak. The origin of this double peak remains to be understood.

The time residual has a bias of 120 ps. For information, 120 ps corresponds to a momentum bias of 4 MeV/c in a 200 MeV/c muon. The width of the distribution is narrower than expected based on the estimated errors. This narrow width may arise from the TOF-tracker cut, which removes particles with TOF01 that is inconsistent from tracker reconstructed momentum, given the muon hypothesis.

6 TOF1 Track Residuals

Residuals from the TOF1 track reconstruction algorithm are also shown. P_z exhibits a bias of 6 MeV/c, which is larger than would be expected from the raw TOF01 bias. The width of the distribution remains to be studied *Rogers: need to fix Pz normalised.* P_x and P_y also exhibit a bias; the source is to be studied.

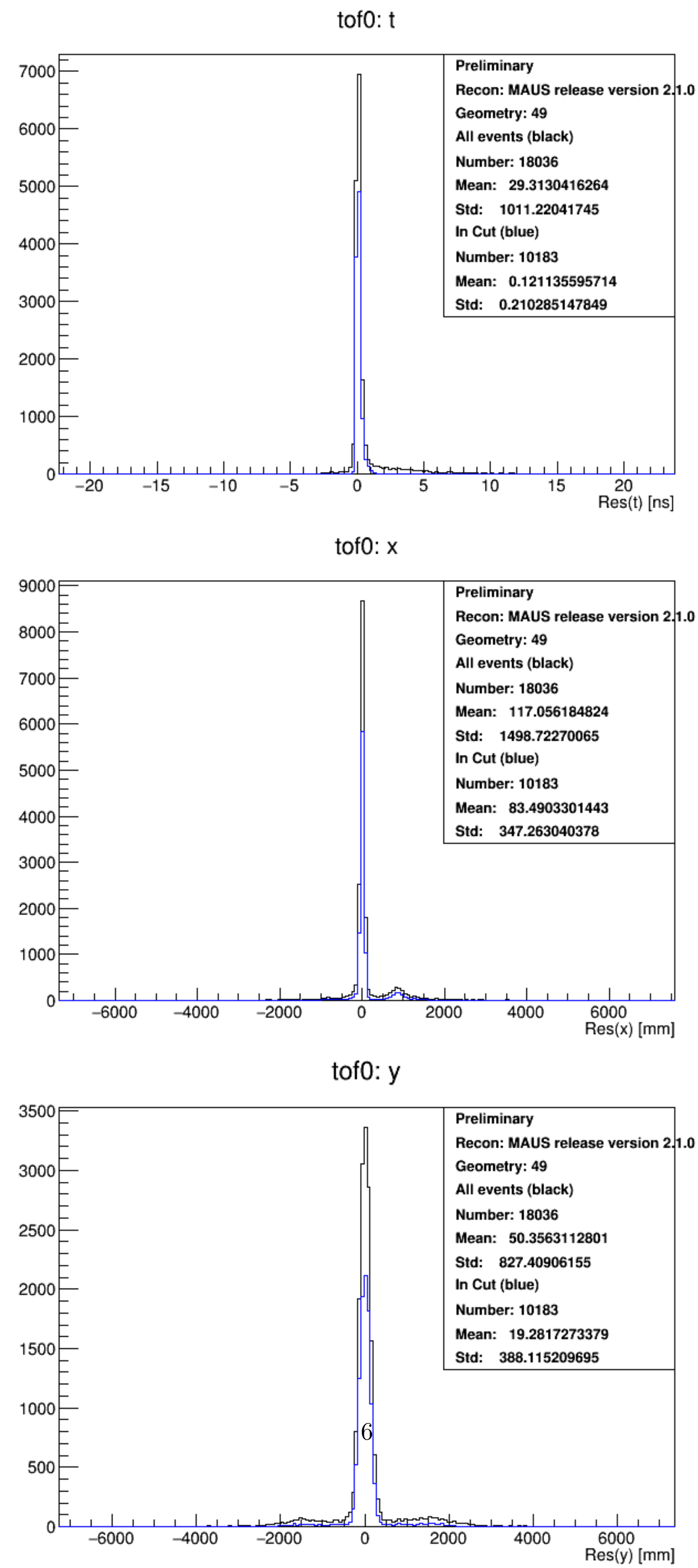


Figure 4: TOF0 residuals.

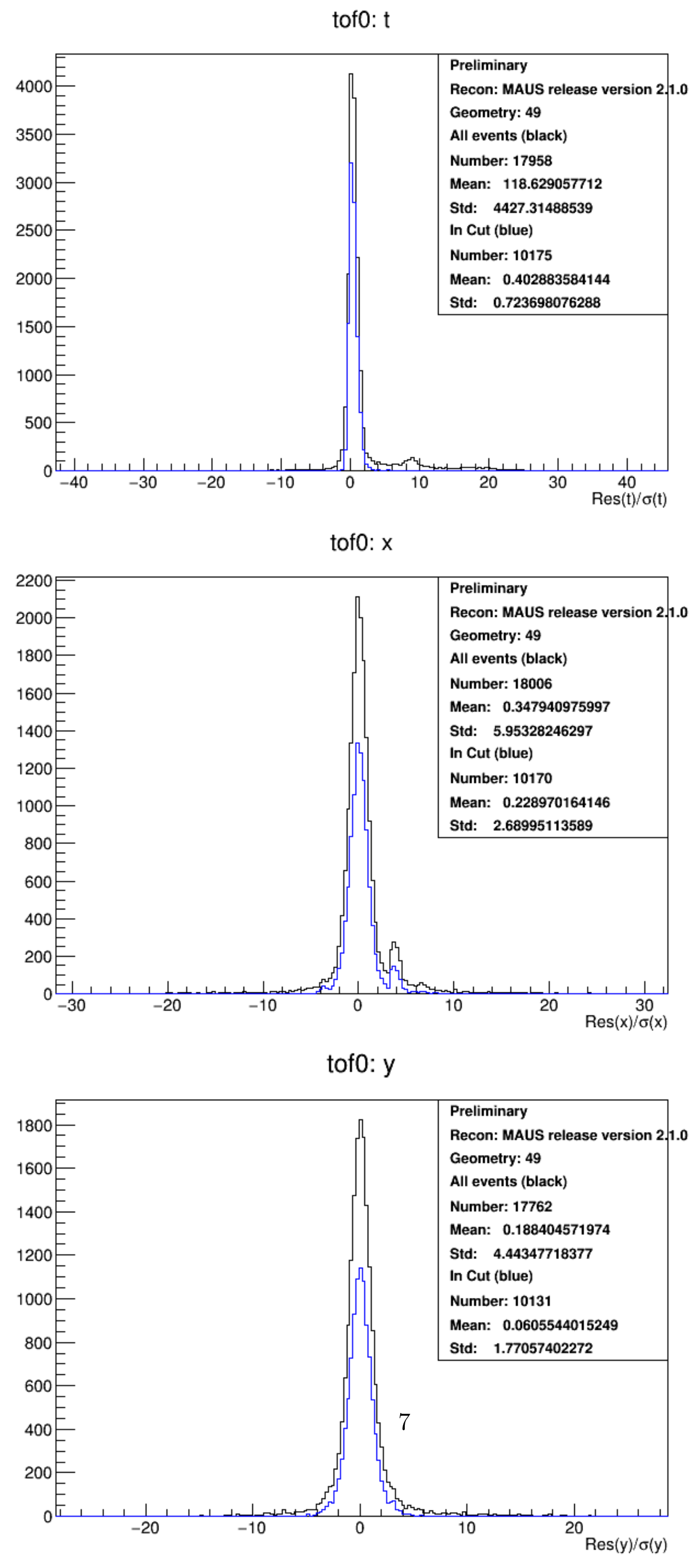


Figure 5: Normalised TOF0 residuals.

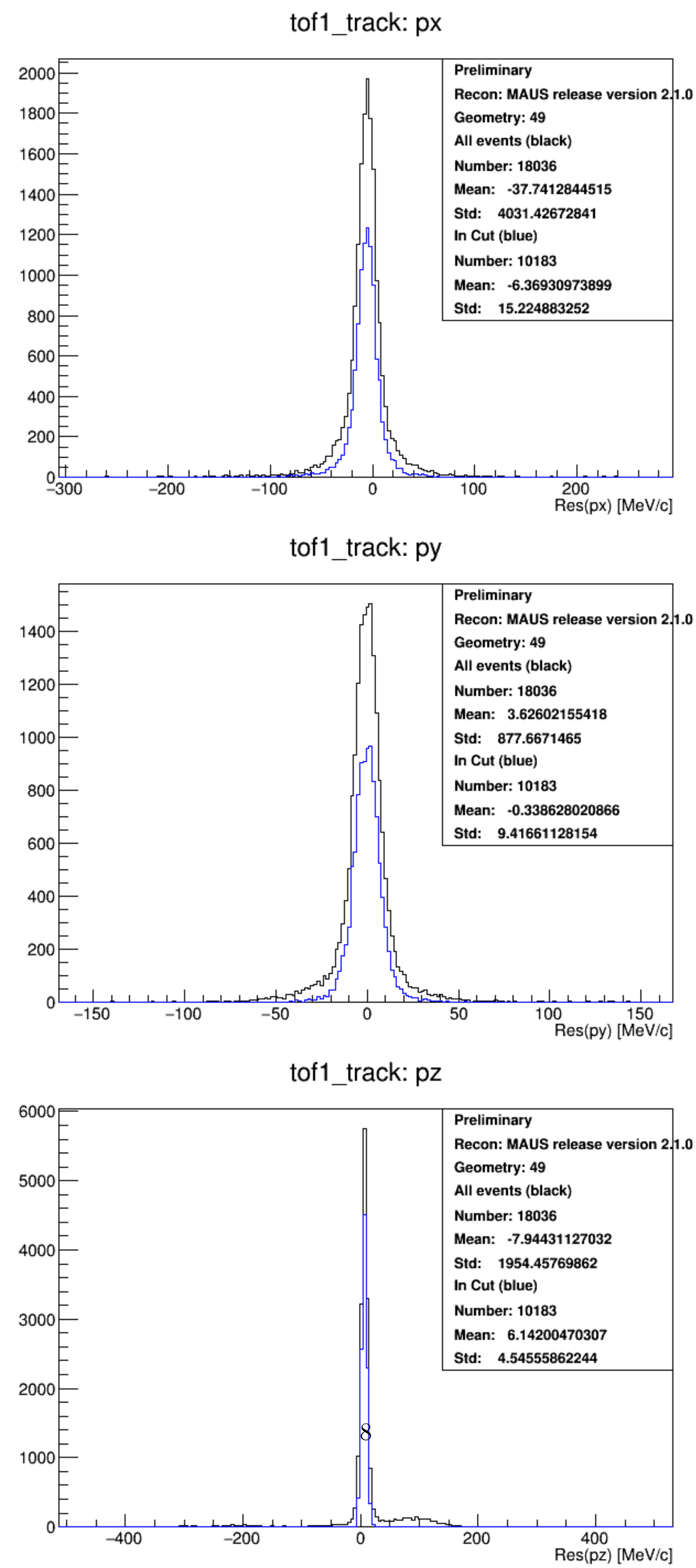
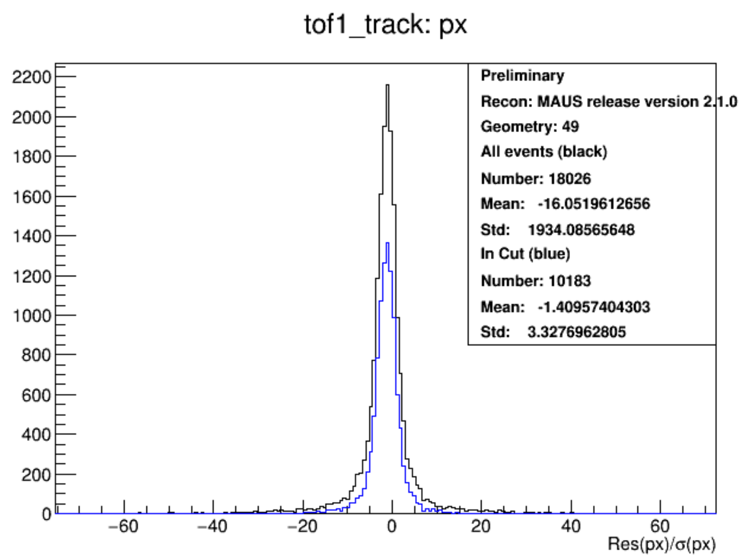
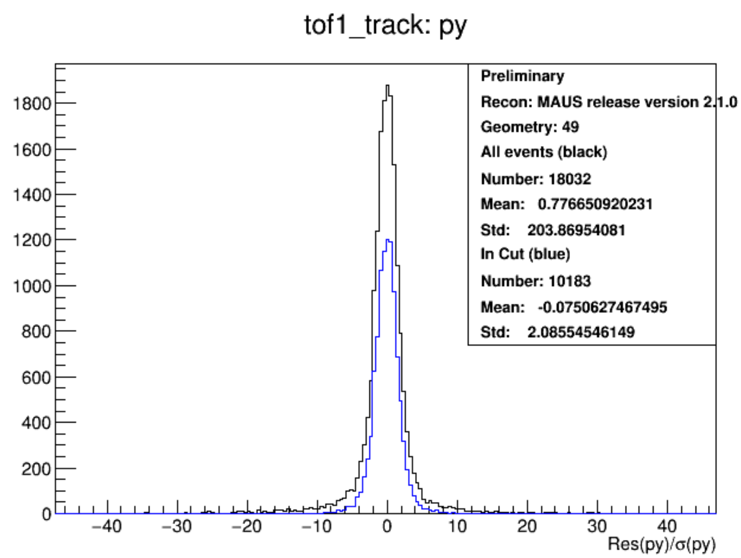


Figure 6: TOF Track residuals calculated at the upstream edge of TOF1.





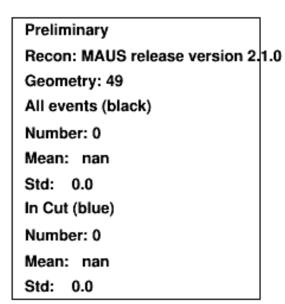


Figure 7: Normalised TOF Track residuals calculated at the upstream edge of TOF1; sorry, the Pz is broken.