Gaussian Kernel Density Estimation (KDE) in MICE

Illinois Institute of Technology

Tanaz A. Mohayai

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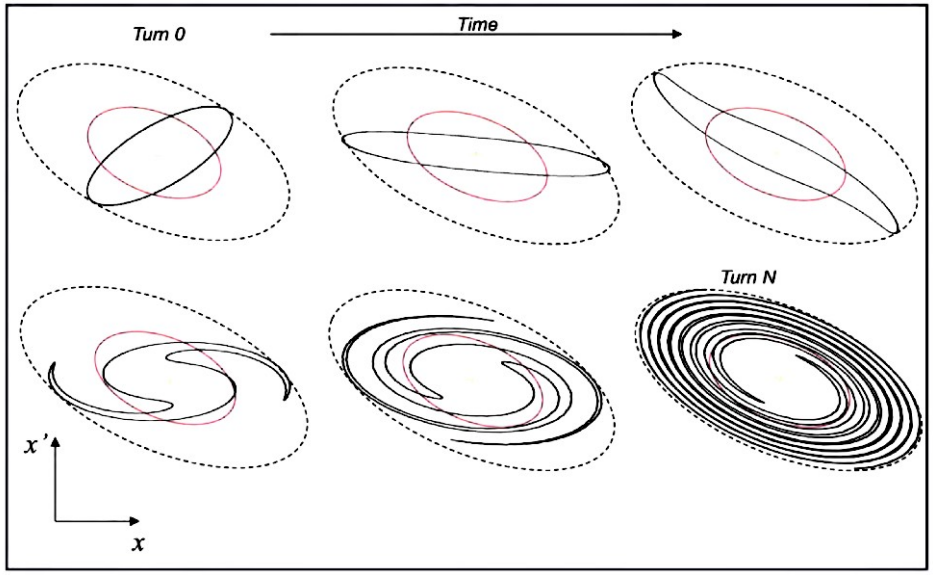
Motivation

- Solenoid beam optics prone to filamentation and other non-linear effects.



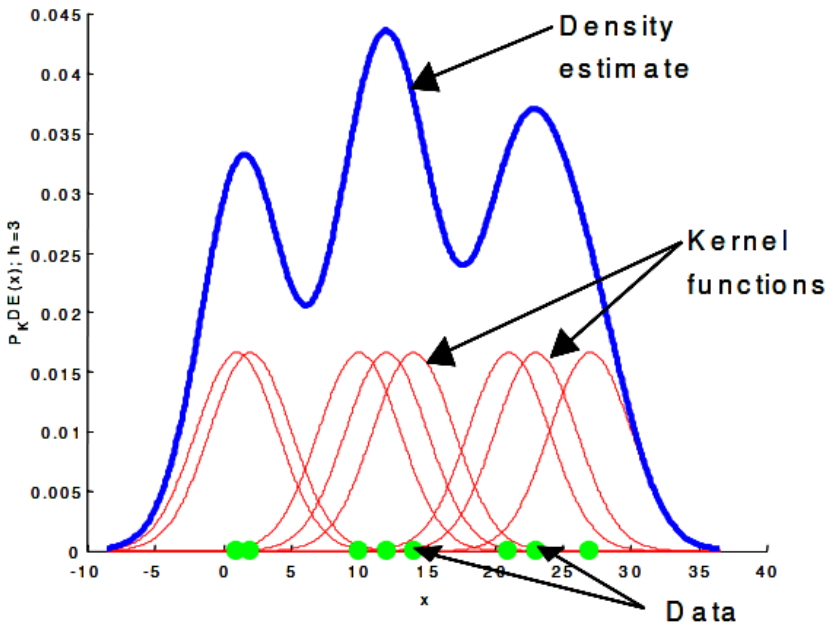
V. Kain, “Beam Transfer and Machine Protection”, CERN

- Non-linearities cause the beam's distorted shape to fill larger ellipse → “apparent” emittance growth.
- Need to study alternative measures of estimating the true phase space volume occupied by the beam as opposed to RMS emittance → Kernel Density Estimation (KDE) can be used.

Background

- KDE → estimates PDF of the particle distribution in phase space using pre-defined kernel functions.
- KDE is a non-parametric DE method, defined as below (n number of points and h smoothing parameter),

$$\hat{f}\left(\vec{x}\right) = \frac{1}{h^d N} \sum_{i=1}^{N} K\left(\frac{\vec{x} - \vec{x_i}}{h}\right),$$



R. Gutierrez Osuna, "Kernel density estimation", CSCE 666 Pattern Analysis, Texas A&M University.

- MICE has ~gaussian beam → PDF estimation using guassian kernel,

$$K\left(\frac{\vec{x} - \vec{x_i}}{h}\right) = \frac{1}{(2\pi)^{\frac{d}{2}}} \exp\left(-\frac{\left\|\vec{x} - \vec{x_i}\right\|^2}{2h^2}\right).$$

Approach

- 2D KDE algorithm routine:

➔ Set up a grid by separately meshing (x, p_x) and (y, p_y).

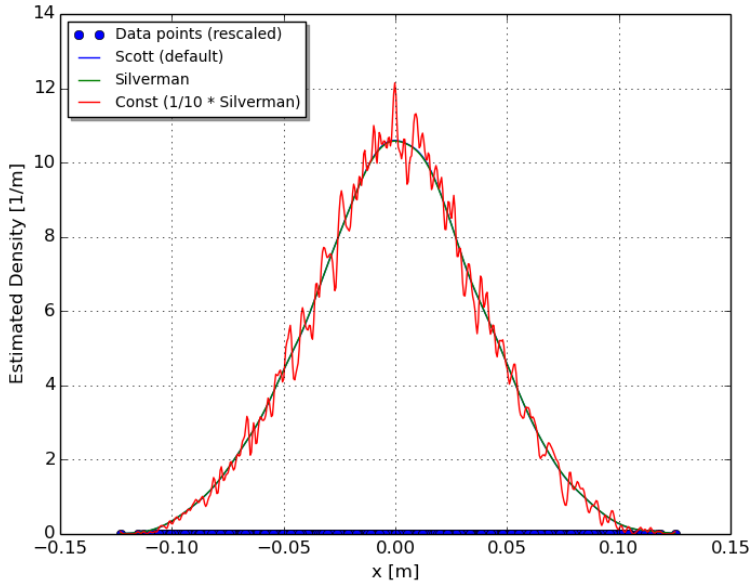
➔ Reshape the grid to (# dimensions, d, # points, n) for KDE evaluation. Stats.gaussian_kde() module in scipy used.

➔ Estimate the probability density functions of reshaped (x, p_x) and (y, p_y) grid using gaussian kernels.

➔ Define bandwidth method (smoothing parameter)

→ Scott's factor is used in this analysis, $n^{-\frac{1}{d+4}}$
→ Other possibility is Silverman factor,

$$(\frac{1}{4}n(d+4))^{-\frac{1}{d+4}}$$



➔ Make a contour plot where contour lines around different levels of the distribution represent constant estimated density.

➔ Calculate the area within the individual contour lines using Green's theorem,

$$A = \frac{1}{2}\int_c xdy - ydx$$

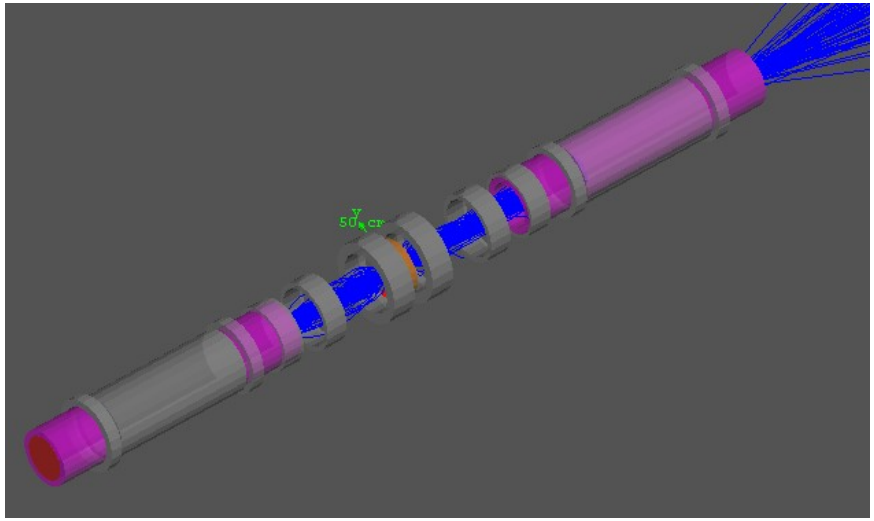
Performance of MICE Step IV with KDE

- Simulation routine:

➔ Beam generation script uses MAUS and xboa routines to produce a beam input file. Script and config files provided by Chris Rogers.

➔ Beam input file is read by Pavel and Ao's MICE Step IV lattice with downstream Match coils. Coil settings from Tim Carlisle's thesis.

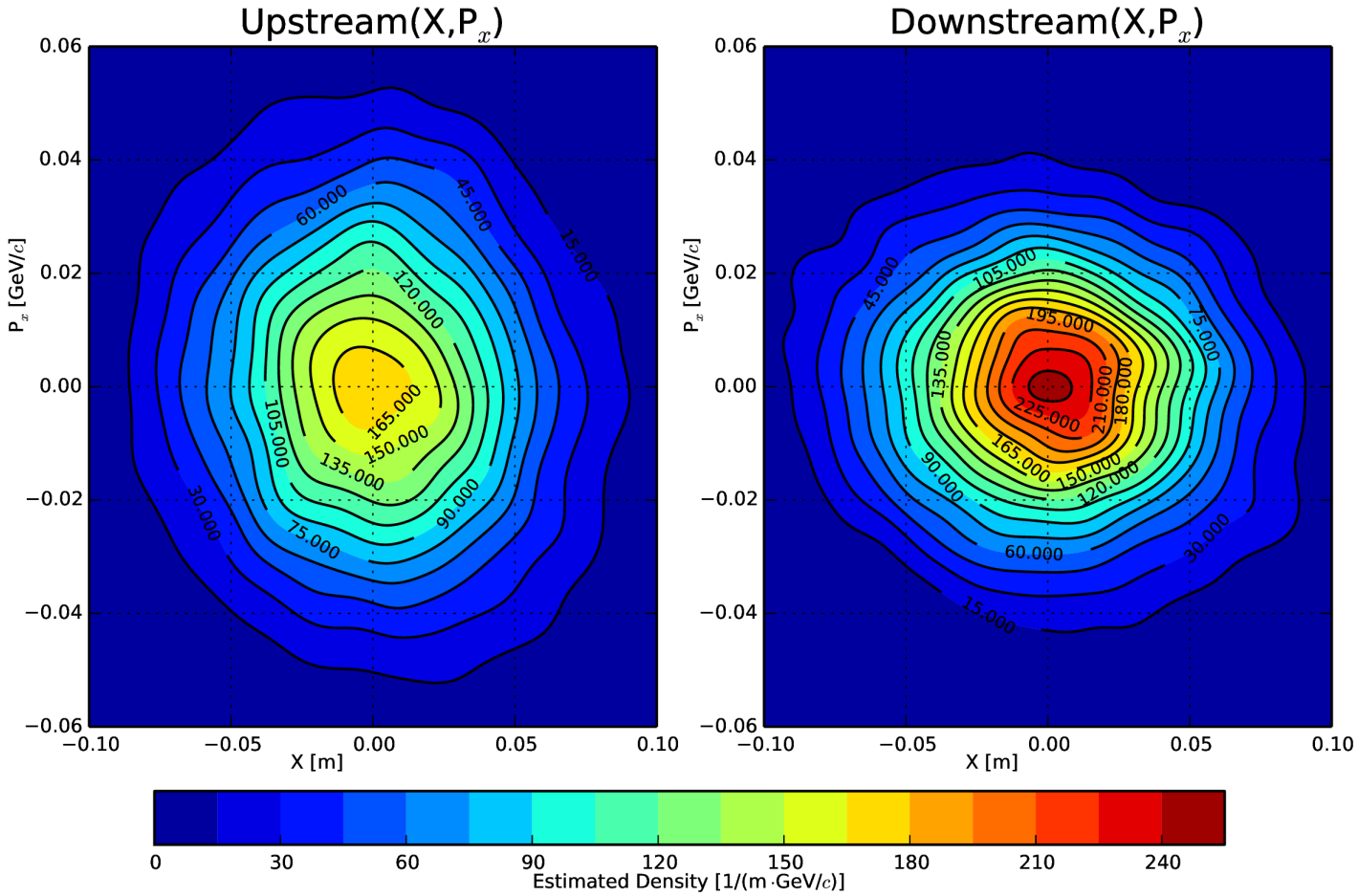
➔ Beam parameters shown in the table,



Simulation Parameters	Values
Number of events	10000
Momentum	140 MeV/c
Emittance	4.2 π mm·rad
Beam type	Gaussian
Number of good muons	9,991

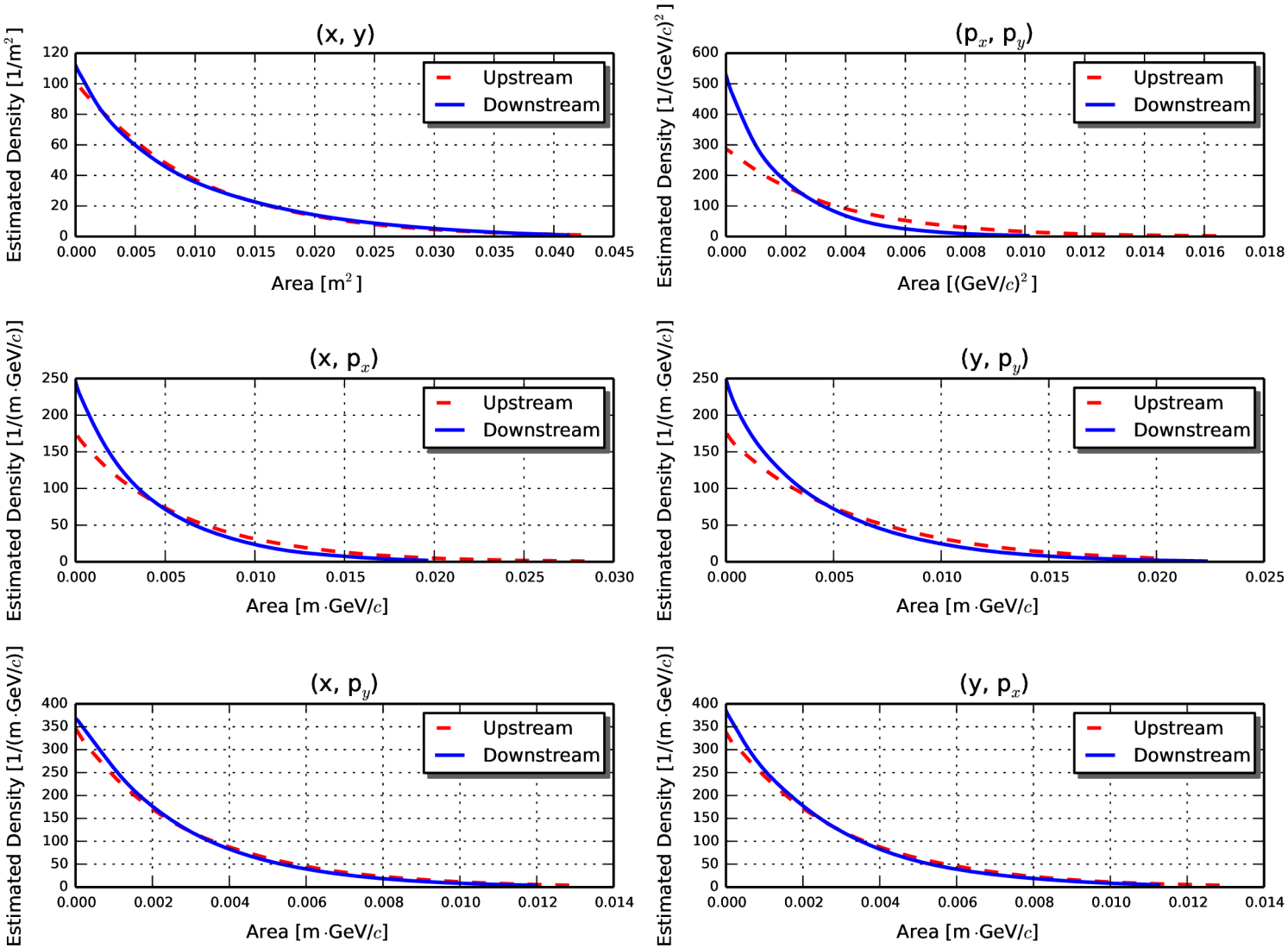
Preliminary MICE Step IV Plots – (x, p_x)

- The cooling performance of the lattice demonstrated as an increase in estimated density from upstream to downstream; better observed for muons closer to the beam center,



Preliminary MICE Step IV Plots – Density vs. Areas within Contours

- For the same areas enclosed within contours, we observe increase in estimated density from upstream to downstream.



Looking Ahead

- We started expanding this analysis to 4D. Phase space volume calculation in 4D under investigation through various methods.
- Examination of the estimated desnsities using different smoothing factors under study.
- Other methods (particle-in-cell) under study.
- Extend the analysis to MICE Demonstration Step.
- IPAC'16 Paper almost ready. We are in the process of preparing it to be emailed to Victoria, hopefully today or tomorrow.