Global Track fitting; Fitting in non-uniform fields

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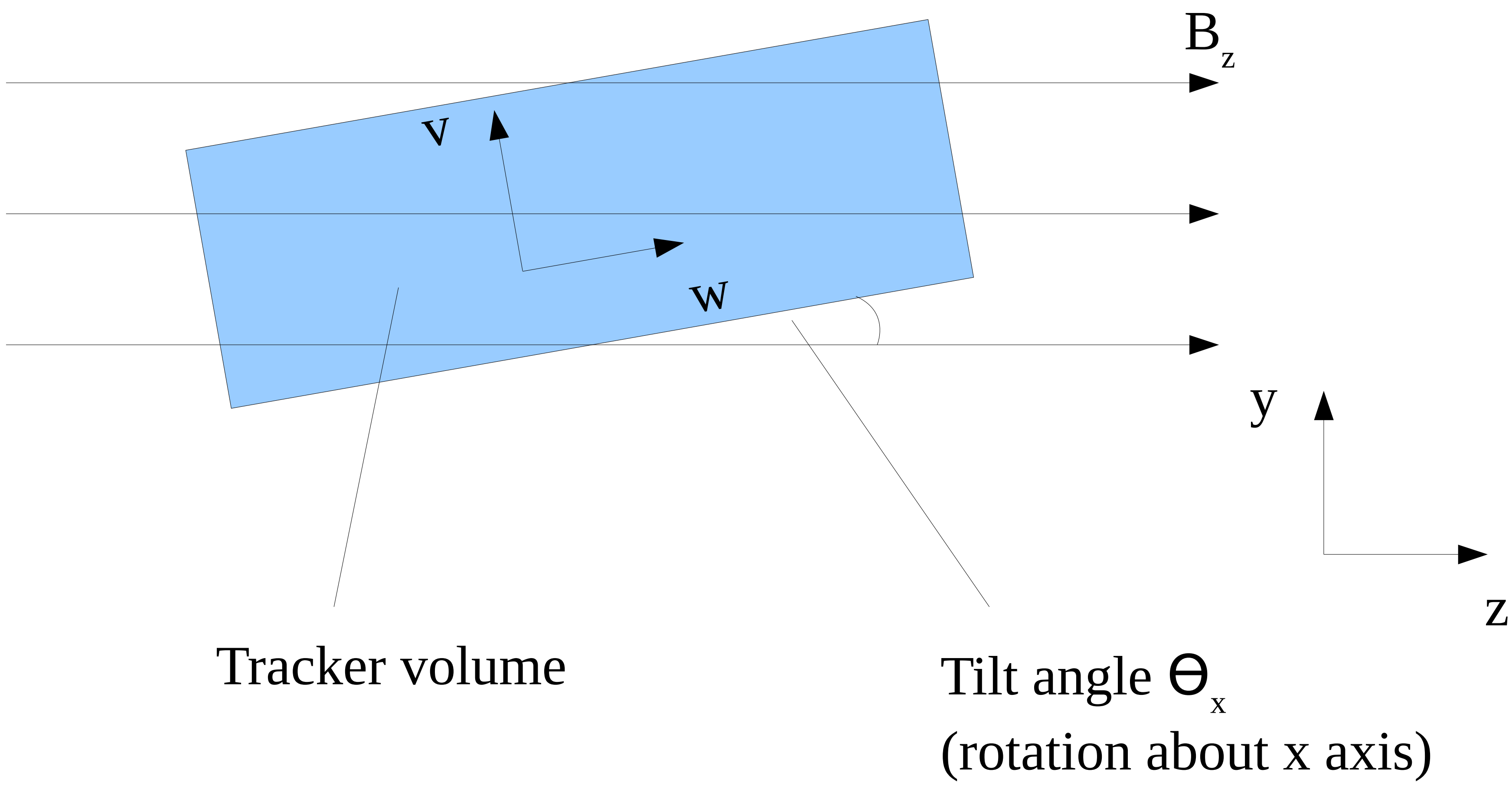
Tracker to Solenoid Alignment

- Aim is to measure the tracker tilt angle wrt solenoid field



Approach

- Can make a cycloid fit to find tracker-to-solenoid tilt
  - Energy loss and field non-uniformity make systematic errors
  - Estimate systematic errors using a massive MC study
- Make a more sophisticated fit that accounts for energy loss and field non-uniformity
- Kalman fit accounts for energy loss
  - But not implemented for non-uniform fields
- Need to do some development work here
- Fit in non-uniform field is a useful algorithm anyway
  - Track fitting in badly trimmed solenoid
  - Global track fitting (using TOF)
  - Generalised (magnet) alignment
  - Systematic error estimation (track matching)
  - Etc etc
- Here
  - Fitting by minimising position residuals
  - Error propagation and improved fitting?

“Minimise Position Residuals”

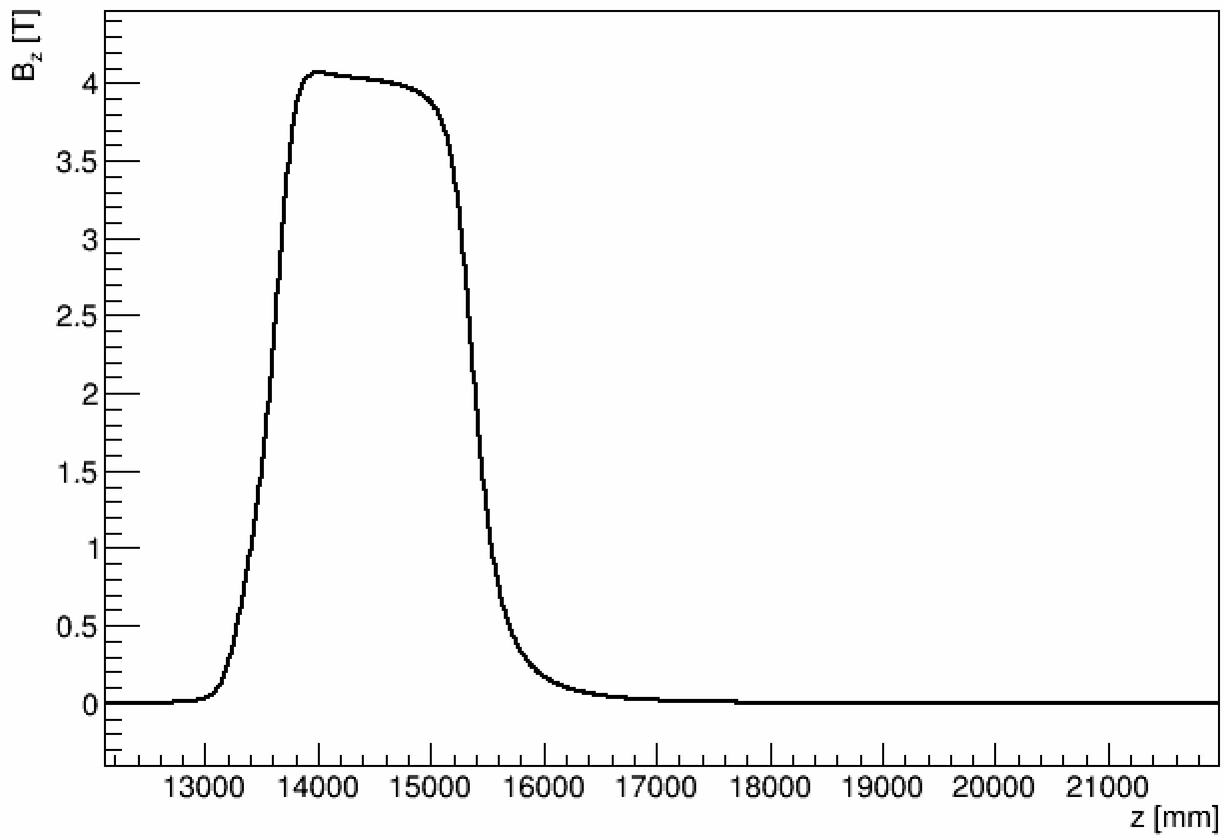
- Look at Run 7475
- Track extrapolation done by integrating equations of motion using RK4
  - No energy loss
  - No stochastic processes
- Use Minuit SIMPLEX routine
  - Vary *x, y, px, py, pz*
  - Score on
    - $\Sigma_{tof}[t(TOF)-t(fit)]^2/0.07^2 + \Sigma_{scifi}[x(scifi)-x(fit)]^2/0.4^2 + \Sigma_{scifi}[y(scifi)-y(fit)]^2/0.4^2$
  - Fit means fitted track extrapolated to the measurement plane
  - Should tweak the parameters a bit to match measured resolution…

“Minimise Position Residuals” (2)

- Require:
  - Exactly one space point in TOF1 and TOF2
  - Exactly one triplet space point in each station in TKU and TKD
  - Can have doublet space points but these are not considered
- Seed:
  - P_z from TOF2- TOF1 assuming muon hypothesis
  - x', y' from TKU station 2 – TKU station 1
  - x, y, z from TKU station 1

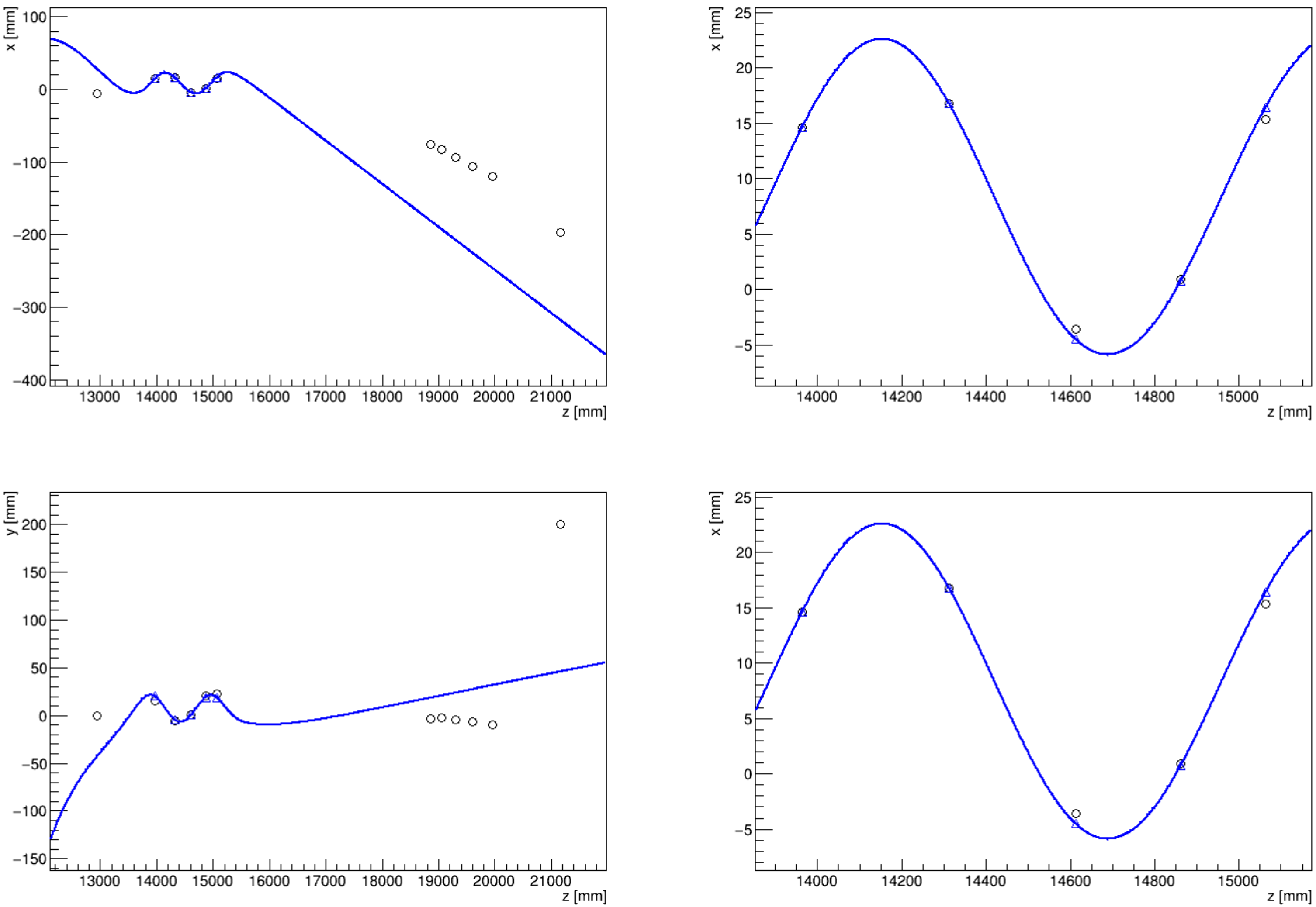
Bz

- B_z taken from Holge Witte field map



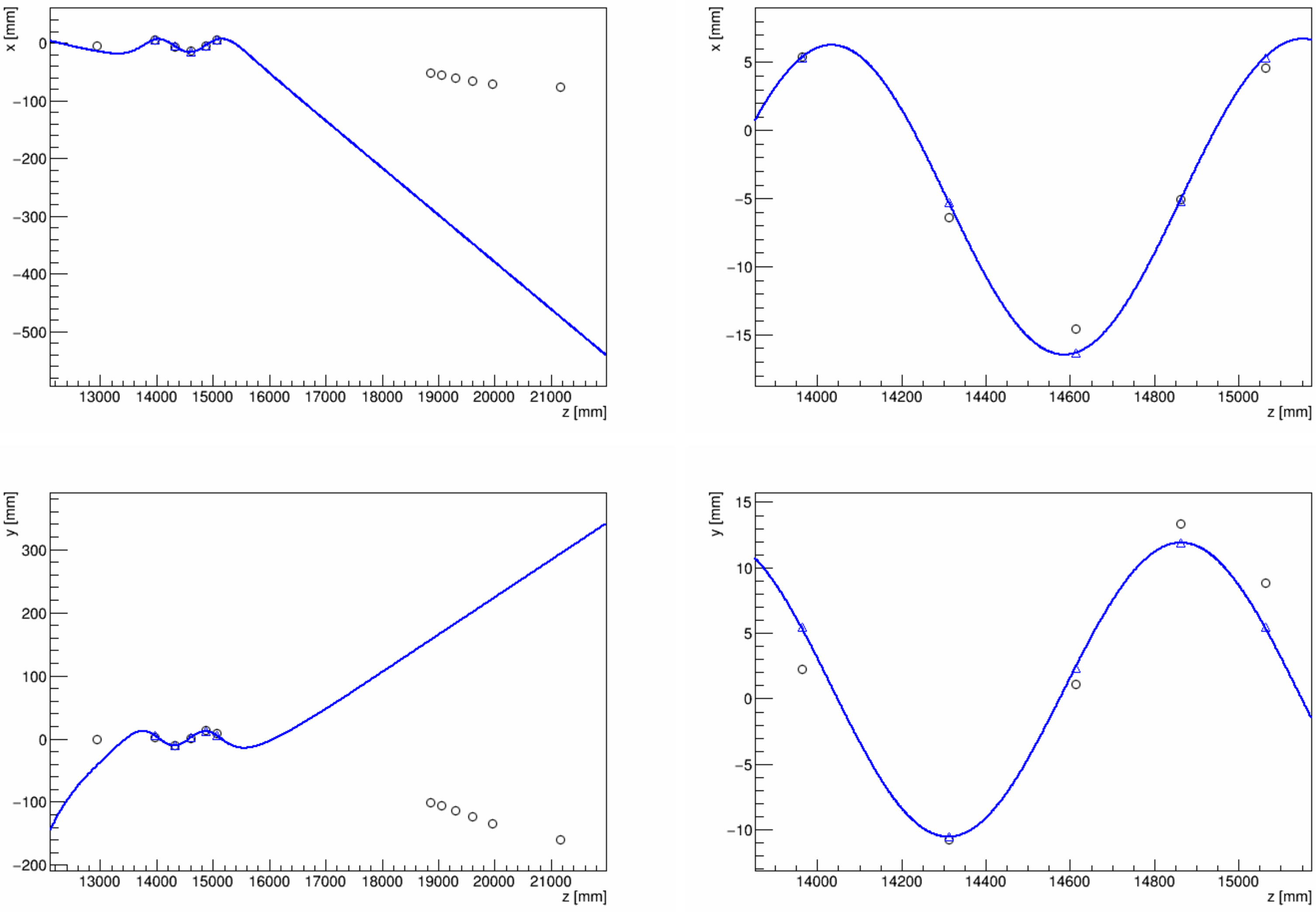
Fit – event display

- First event in run 7475 matching cuts:



Fit – event display

- Second event in run 7475 matching cuts:



Comments

- Anecdotally, looks like the fit is essentially working
  - Black circles are space points
  - Blue line is the fitted track
  - Blue triangles are points on fitted track
- Anecdotally, matching with external detectors looks poor
  - Note these were NOT included in the fit
- Generalised fit with external detectors requires propagation of errors between detectors
  - Scattering is the dominant source of uncertainty
  - Scattering is an uncertainty on the propagated track, hence need to propagate errors
  - Also required for proper estimation of e.g. chi2
  - Required to quantify “matching with external detectors looks poor”

Error Propagation thru Fields

- We have a trajectory with accelerator phase space vector (Kalman state vector) $\underline{u}_{in}$ at a given measurement plane and $\underline{u}_{out}$ at the next measurement plane
- Consider accelerator transfer matrix (Kalman propagator) **M**, defined by

$$\underline{u}_{out} + \mathbf{M}\,\underline{du}_{in} = \underline{u}_{out} + \underline{du}_{out}$$

- $\underline{u}$ is the and $\underline{du}$ is a small deviation from the vector
  - This is the first term in a Taylor series
- **M** is found by differentiating the equation of motion for $\underline{u}$
- Quote

$$\underline{F} = d\underline{p}/dt = q\,\underline{v}\ \mathbf{x}\ \underline{B}$$

- Then

$$d\underline{p}/dz = q\,d\underline{x}/dz\ \mathbf{x}\ \underline{B}$$

- Also

$$d\underline{x}/dz = \underline{p}/p_z$$

- Derivatives of this wrt $\underline{u}$ give the analytical transfer matrix...

Error Propagation thru Fields (2)

- Consider the accelerator beam ellipse (covariance matrix) **V** with elements

$$V_{ij} = \langle u_i u_j \rangle$$

  and centroid $\underline{u}$
- Then error matrix **V** propagates like

$$\mathbf{V}_{\mathbf{out}} = \mathbf{M}\,\mathbf{V}_{\mathbf{in}}\,\mathbf{M}^{\mathbf{T}}$$

- I want to integrate V, so I want d**V**/dz = [**V**(z+dz)-**V**(z)]/dz
- For small dz, **M** ~ **1**+**dM** so

$$\mathbf{V}(z+dz)-\mathbf{V}(z) = \mathbf{dM}\,\mathbf{V}\,\mathbf{dM}^{\mathbf{T}} + \mathbf{dM}\,\mathbf{V} + \mathbf{V}\,\mathbf{dM}^{\mathbf{T}}$$

- Note that this is a specialisation to Lorentz force law for the generalised problem of error propagation between two (sets of) variables using Jacobian
- The algebra is quite fiddly
- I work in coordinate system $\underline{u}$ = (x, y, t, px, py, total energy)

M: comparison with numerical (1)

- Can compare analytical form for **M** with numerical form for **M**
  - Take numerical derivative
  - On axis case:

t	**x**	**y**	**z**	**energy**	**px**	**py**	**pz**
0	0	0	5	226.194	0	0	200
Analytical transfer matrix							
	0	0	0	-4.65E-006	0	0	
	0	0	0	0	0.005	0	
	0	0	0	0	0	0.005	
	0	0	0	0	0	0	
	0	0	-0.00614736	0	0	0.00131658	
	0	0.00614736	0	0	-0.00131658	0	
Numerical transfer matrix							
	0	0	0	-4.65E-006	0	0	
	0	0	0	0	0.005	0	
	0	0	0	0	0	0.005	
	0	0	0	0	0	0	
	0	0	-0.00615725	0	0	0.00131611	
	0	0.00615725	0	0	-0.00131611	0	

M: comparison with numerical (2)

- Can compare analytical form for **M** with numerical form for **M**
  - Take numerical derivative
  - off axis case:

t	x	y	z	energy	px	py	pz
0	1	2	5	307.837	200	60	200
Analytical transfer matrix							
	0	0	0	-2.28E-005	2.57E-005	7.70E-006	
	0	0	0	-0.00769592	0.01	0.0015	
	0	0	0	-0.00230878	0.0015	0.00545	
	0	0	0	0	0	0	
	0	0.000806937	-0.0042974	-0.000619458	0.000402459	0.00146227	
	0	0.00379494	-0.00423267	0.00206486	-0.00268306	-0.000402459	
Numerical transfer matrix							
	0	0	0	-2.28E-005	2.57E-005	7.70E-006	
	0	0	0	-0.00769595	0.01	0.00150004	
	0	0	0	-0.00230867	0.0014999	0.00544998	
	0	0	0	0	0	0	
	0	0.000811811	-0.00430388	-0.000619319	0.000402379	0.00146193	
	0	0.00379078	-0.00424139	0.00206446	-0.0026825	-0.000402496	

Fit – event display

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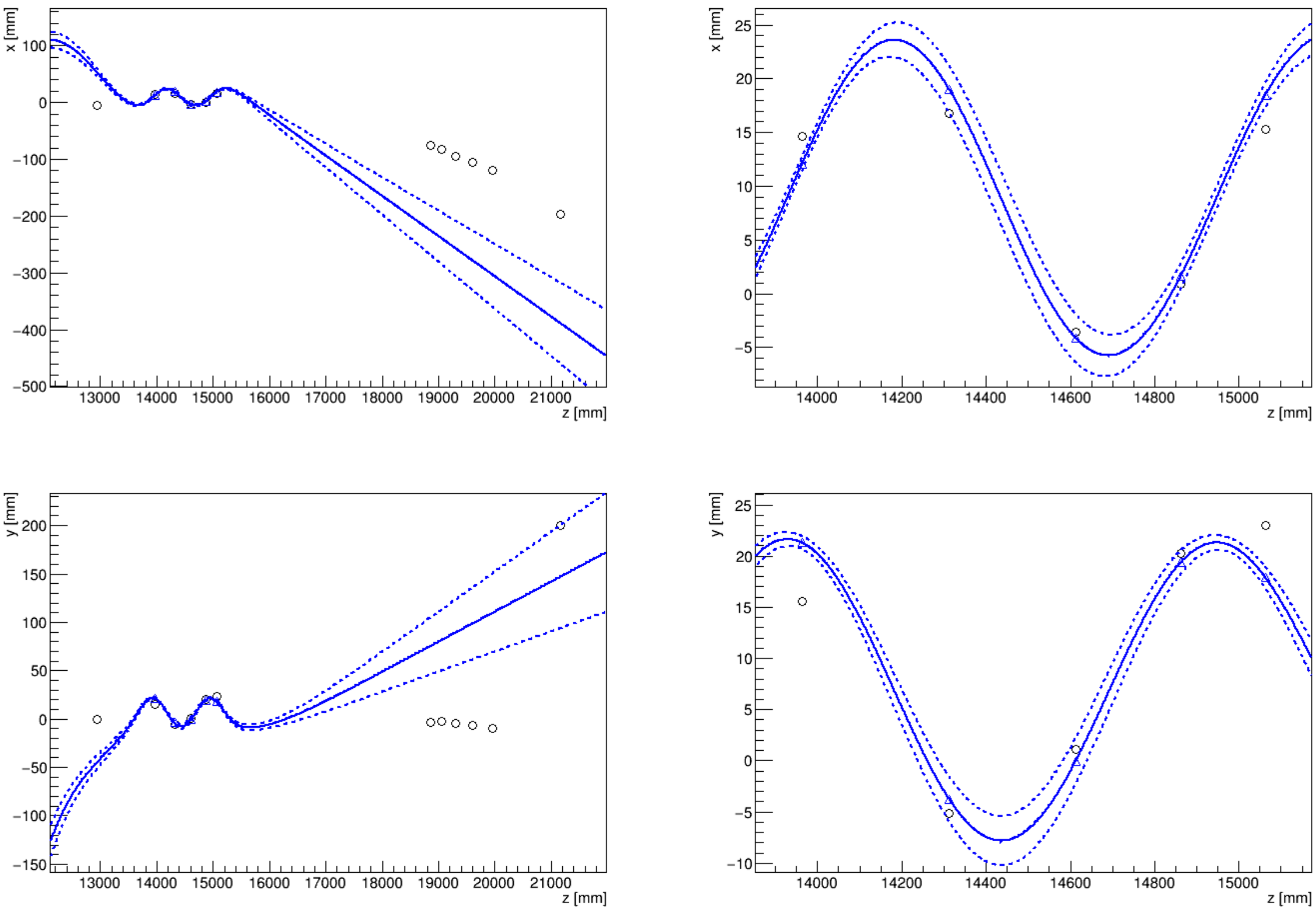

Fit – event display (2)

- Note that minuit does not do errors how I want
  - I put on some “representative” errors based on Kalman track fits
- Code/scripts are in lp:~chris-rogers/maus/tracking_errors
  - Certainly not production ready
  - Error propagation stuff is close (CM44?)
  - Maths notes are attached to #1600 – needs to be typed up
- To do:
  - Energy loss
  - Multiple Coulomb Scattering
  - Energy straggling
  - Tidy error propagation code and push
  - Proper fitting algorithm
    - Modify SciFi Kalman?
    - Something else?
  - Write up