STATISTICAL WEIGHTING OF THE MICE BEAM

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Abstract

Conventionally only average properties such as means and variances of charged particle beams are measured. Such a technique is limited in that it is challenging to measure moments beyond the second and certain correlations are difficult to measure. In the Muon Ionisation Cooling Experiment (MICE) [1], the beam rate is sufficiently low that particles pass singly through the accelerator and measurements can be made of the position, time (relative to RF phase) and momentum of individual particles. This makes a number of new analysis tools available. In this paper one particular tool is studied; the ability to select an input beam.

SELECTION OF A BEAM FOR MICE

In MICE, muons are produced by pions decaying upstream of the experiment. They then traverse a set of detectors that measure the type and phase space coordinates of the particles, a cooling apparatus that performs some cooling and then another set of detectors that remeasure the type and phase space coordinates of the particle. MICE measures the cooling performance by comparing the input distribution of particles with the output distribution of particles. A principal measure of cooling is the change in beam emittance, which is some function of the second moments of the beam.

In MICE, the cooling performance is dependent on the input distribution of particles. In particular, for good cooling performance the beam must be aligned and matched to the magnetic lattice such that the beam centre travels along the cooling channel axis and the covariance matrix is periodic over the length of a cooling cell. This places a constraint on the beam first and second moments. In addition, it has been observed that the MICE cooling performance is quite dependent on certain third moments such as the amplitude-momentum covariance of the beam.

Certain aspects of the input beam can be controlled by the MICE muon beamline. This beamline has been designed to generate beams with a variety of different transverse emittances matched to the MICE lattice in transverse phase space. The beamline is expected to provide only limited control over longitudinal phase space. In particular, the muon beamline is expected to produce a beam with a larger energy spread than is desirable, with undesirable correlations between energy and transverse coordinates and with a flat distribution in time [2].

Hence, in order to achieve the best possible cooling performance, it is expected to be necessary to statistically weight events in order to choose a beam that is matched sufficiently for the experiment to observe and measure cooling to high precision. Events that sit in an area of phase space that is statistically depleted relative to the matched distribution would receive a larger weight; and those that sit in an area of phase space that is statistically enriched relative to the matched distribution would receive a smaller weight. This weighting must be performed in six dimensions due to coupling between transverse phase spaces due to solenoidal fields and coupling between transverse and longitudinal phase space due to non-linearities in the beam transport and dispersion introduced by the beamline. This is difficult as traditional binning techniques fail for high dimensions due to the sparseness of phase space, even for beams with quite high statistics.

In this note, I present two methods by which statistical weights can be applied to events according to their position in phase space in order to select the beam moments to match some desired distribution. I then simulate a beam passing through MICE and show the change in emittance for the weighted and unweighted beams.

WEIGHTING BY BEAM MOMENTS

In accelerator physics, evolution of a beam is usually expressed in terms of some perturbation theorem about some reference trajectory, which can be used to describe the evolution of the beam moments [3]. The n^{th} order transfer map couples m^{th} moments to $(m+n-1)^{th}$ moments, so higher order terms in the perturbation couple lower moments to progressively higher moments. These higher order terms can produce undesirable effects such as emittance growth. Such a formalism lends itself to using a moment-based technique to weight the beam.

Consider some input distribution of events $f(\vec{x})$ and some desired output distribution $g(\vec{x})$ in a multidimensional phase space described by phase space vector $\vec{x}$. I apply a statistical weighting to each event given by a polynomial function $w(\vec{x})$. I weight with the multi-dimensional polynomial

$$\begin{aligned} w(\vec{x}) &= 1 + \sum a_{i_1}(x_{i_1}) + \sum a_{i_1 i_2}(x_{i_1} x_{i_2}) + \\ &+ \sum a_{i_1 i_2 i_3}(x_{i_1} x_{i_2} x_{i_3}) + \ldots \end{aligned} \quad (1)$$

such that

$$\begin{aligned} g(\vec{x}) &= N(1 + \sum a_{i_1}(x_{i_1}) + \sum a_{i_1 i_2}(x_{i_1} x_{i_2}) + \\ &+ \sum a_{i_1 i_2 i_3}(x_{i_1} x_{i_2} x_{i_3}) + \ldots) f(\vec{x}) \end{aligned} \quad (2)$$

For convenience of notation, I denote moments of the distribution $f(\vec{x})$, $< x_{i_1} x_{i_2} \ldots x_{i_n} >_f$ by $V^f_{i_1 i_2 \ldots i_n}$.

An n^{th} moment of the function $g(x)$, $V^g_{j_1\ldots j_n}$, can be written as

$$\frac{V^f_{j_1\ldots j_n} + \sum a_{i_1}(V^f_{i_1 j_1\ldots j_n}) + \sum a_{i_1 i_2} V^f_{i_1 i_2 j_1\ldots j_n}) + \ldots}{1 + \sum a_{i_1}(V^f_{j_1\ldots j_n}) + \sum a_{i_1 i_2} V^f_{j_1\ldots j_n} + \ldots)}. \quad (3)$$

Rearranging leads to

$$\begin{aligned} V^g_{j_1\ldots j_n} - V^f_{j_1\ldots j_n} &= \sum_{i_1\ldots i_m} a_{i_1\ldots i_m}\Big(V^f_{i_1\ldots i_m j_1\ldots j_n} \\ &- V^f_{i_1\ldots i_m} V^g_{j_1\ldots j_n}\Big). \end{aligned} \quad (4)$$

This is actually a linear problem. Defining vectors $\vec{u}$ and $\vec{a}$ with elements $u_i = V^g_{i_1\ldots i_n} - V^f_{i_1\ldots i_n}$ and $a_i = a_{i_1\ldots i_n}$ and defining the matrix $\mathbf{M}$ with elements $M_{ij} = \Big(V^f_{i_1\ldots i_m j_1\ldots j_n} - V^f_{i_1\ldots i_m} V^g_{j_1\ldots j_n}\Big)$. Then 4 reduces to

$$\mathbf{M}\vec{a} = \vec{u}, \quad (5)$$

which can be solved for the polynomial coefficients $a_{i_1\ldots i_n}$.

I show contours of an unweighted distribution of particles and the distribution after weighting using this algorithm in Figure 1. 10000 particles were sampled from a two dimensional multivariate Gaussian distribution with normalised emittance ϵ_x 15 mm, β_x 334 mm and α_x 0. Moments were weighted up to tenth moments to a distribution with normalised emittance ϵ_x 4.2 mm, β_x 260 mm and α_x -0.75. The algorithm is successful in producing a beam with the desired moments. In particular, the optical functions and emittance of the weighted beam and the target distribution are the same to good precision.

In general this algorithm is quite successful. However, for higher dimensions the time taken to calculate the beam moments can be long, the algorithm can generate negative weights and for higher moments the tails of the distribution can dominate the calculation, eventually leading to numerical precision errors. The latter two problems may be soluble by choosing a better set of functions for weighting.

VORONOI TESSELATION

It is possible to tesselate a distribution of particles by defining a cell for each particle, the region closest to a certain particle. This is known as a Voronoi tesselation [4]. By calculating the volume of each Voronoi cell, we can then calculate the phase space density for each particle in the distribution. In Figure 2 I show the Voronoi tesselation of a set of particles in some two dimensional space.

After calculating the density associated with each particle in the distribution, it is then possible to weight the beam by requiring this density to be that appropriate for some target distribution. Such a weighting is show in Figure 3. Here, the Voronoi tesselation for a set of points was found in two dimensions using the QHull code [5]. The effect of the edge of the beam was reduced by defining the fiducial phase space volume of the detector, in this case taken to be an upright ellipse with width in P_x of 100 MeV/c and

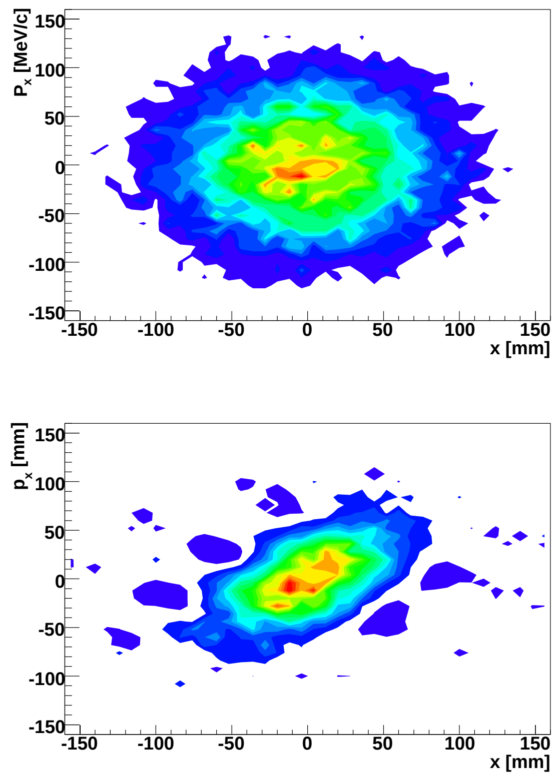

Figure 1: 10000 particles sampled from a Gaussian distribution, before (top) and after (bottom) statistical weights are applied by the moment method.

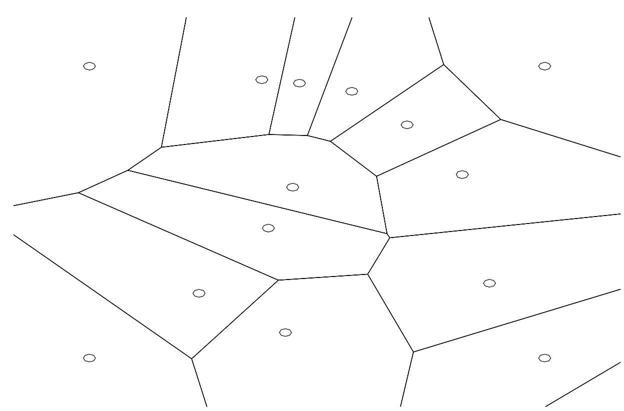

Figure 2: An example of a Voronoi tesselation.

in x of 150 mm. Particles were added in an ellipse at radius 10 $\%$ larger than this fiducial volume, and the volume of the Voronoi cells were calculated using these additional particles. Subsequently particles outside the fiducial volume were discarded, including these additional particles. This prevents particles on the distribution boundary receiving very large weightings.

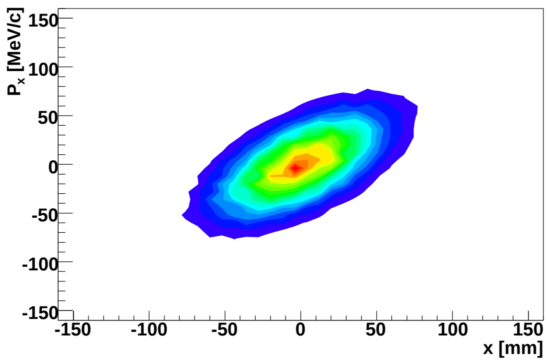

Figure 3: The distribution weighted now using the Voronoi algorithm.

While this technique is very useful, it is constrained by the amount of computer memory required to record the teselation in a high dimensional space. This typically constrains the number of particles or phase space dimensionality that can be used in the tesselation. Alternative Voronoi algorithms may reduce this requirement. In addition, some particles can sit in regions of rather high volume and so have large statistical weights applied. This may lead to greater statistical errors in any cooling measurement.

SIMULATED BEAM

Two beams were simulated passing through the MICE cooling channel using the G4MICE package [6], one with a well matched beam and the other with a significant mismatch introduced. It is hoped that the mismatch due to the MICE beamline will not be as severe as that shown. The transverse optical β function and change in transverse emittance of the beam is shown. These calculations were repeated for the mismatched beam with additional statistical weights added to give the beam the distribution of a matched beam using the moment technique outlined above. The cooling performance of the weighted beam is close to that of the matched beam while the mismatched beam is actually heated.

CONCLUSIONS

Two algorithms for choosing statistical weights to make a distribution of particles appear like an alternative distribution have been shown. In MICE, this will enable the measurement of cooling despite a mismatch or misalignment of the input beam that would naturally worsen the actual cooling performance. These algorithms have been used to weight a distribution of particles and its effect on a Monte-Carlo simulation of the cooling distribution has been demonstrated to enable the beam to be rematched.

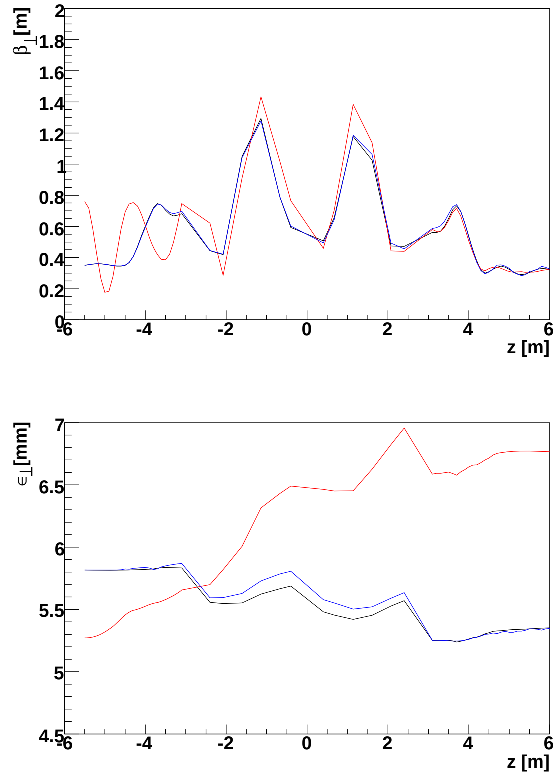

Figure 4: Transverse β function and cooling performance of a (black) matched beam, (red) mismatched beam and (blue) weighted mismatched beam.

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