Issues related to MAUS Geometry implementation

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Conceptualization

Statement of Problem

For a physical object with an arbitrary set of reference positions, $\vec{p}_i$, in the model coordinate system there is a set of measurements matched to those positions, $\vec{q}_i$, in the same coordinate system. We want to define a rotation, $(\theta_x, \theta_y, \theta_z)$, and translation $\vec{k} = (k_x, k_y, k_z)$ that matches $\vec{p}_i$ to $\vec{q}_i$.

- Mathematically this is the system of equations:

$$\vec{q}_i = M\vec{p}_i + \vec{k}$$

where M is the matrix defined by the product of rotation matrices

$$M = R(\theta_x)R(\theta_y)R(\theta_z)$$

- Rotation matrices are defined as

$$R(\theta_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{pmatrix}$$

$$R(\theta_y) = \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{pmatrix}$$

$$R(\theta_z) = \begin{pmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Need to check that this is consistent with GEANT4 rotations.

Implementation

- Consists of a least squares fit of six variables to three or more survey points.
- Will require a new python function (class maybe) to fit for rotations and translations given survey information.
- Once the fit has been done, results can be added to "Maus_Information.xml" for correction to geometry, in the case of detectors or added to model in the case of other elements.
- Possibility that survey and model do not use the same coordinate system
  - Implement a global transformation for this possibility
  - Should this require a global fit?

Algorithm

1. Extract reference points: $\vec{p}_i$
2. Extract data points: $\vec{q}_j$
3. Enforce a suitable number of data points
   - i.e. if $i \in \{0, N\}$ and $j \in \{0, M\}$ $N = M$ and $N > 6$
4. Sort $\vec{q}_j$ to match $\vec{p}_i$ using a χ^2 statistic
5. Fit $\vec{q}_j$ to $\vec{p}_i$ by minimizing a total χ^2
   - Parameters are translations: $\vec{k}$ and rotations $\vec{\theta}$.
6. Write positions and rotations to geometry files.