Journal Editor

We would like you to aim for ~3,500 words; it is at 4,100 words at present.

The formal definition of quantities under study was moved to the methods section.

Add a few words to the introduction … to emphasize the size...

The third paragraph in the introductory material was adapted to emphasize this point.

Try to keep the number of references to below 50 (or 55 at a push)

Some of the introductory references were removed while other material was moved to the methods section, resulting in 50 references.

Fully referenced summary paragraph

The abstract was rewritten to try to better match the formatting outlined in the example.

Title – our usual limit is 75 characters (including spaces)

The title length was reduced.

Supplementary figure should be provided in the 'Extended Data' format

The supplementary figure has been put in "Extended Data" format

Subheadings - … we strongly advise the use of subheadings ...

Subheadings were introduced.

Referee 1

(1) I do not understand the statement, "The correction is described in the Methods section." The draft of the manuscript that I read did not show any sections. Should this say, "... the Methods section of [some citation]" ?

The methods section is placed at the end of the references, on page 14 of the manuscript.

(2) I am surprised that the paper does not attempt to quantify the amount of cooling or density increase. I realize that beam loss would complicate the analysis, but can't the authors find a way to quantify the amount of density increase in the beam core? I would expect this to be possible based on the data that were used to produce Figures 3, 4, and 5.

A table and suitable text was appended to the end of the methods section.

Referee 2

In Figs. 3, the scraping effects are significant, more significant even than the cooling, while the equilibrium emittance of the cooling channel is close to the initial emittance. Indeed, the final equilibrium emittance looks very close to what one could have achieved with scraping alone. It is a bit surprising to me that the experiment was conceived in this way, and that there is not more

margin between the equilibrium emittance and the onset of scraping. Perhaps this situation deserves a brief explanation or comment. Why is there so little separation between the amplitude of scraping and the equilibrium emittance in the cooling channel, and, also, how could, or would, a real ionization cooling system for a collider achieve an orders-of-magnitude smaller emittance compared with this demonstration?

The failure of the superconducting lead in a spectrometer solenoid matching coil and the consequent compromise in optics and acceptance is noted at line 172-177. It is noted on line 180-184 that better cooling lattices are available.

The technique of chaining many cooling cells together to achieve an extended ionization cooling system is described on line 90.

In the abstract, line 2 "Such beams have the potential … "-> "Such beams would have the potential"

The correction was adopted.

Line 21, "Muon beams are created"->"Muon beams can be created" (since other methods and proposals for muon production exist)

The correction was adopted.

Line 32" "High-brightness muon beams have not yet been produced at energies comparable to state-of-the-art electron and proton beams" – This sentence slightly puzzles me. Have very bright muon beams been produced at other energies?
What is the definition of a high-brightness muon beam?

High-energy muon beams are available but not at sufficient brightness for muon collider or neutrino factory applications. The sentence was moved to the next paragraph and the wording was clarified.

Line 41, "making collisions possible" -> "potentially enabling collisions "; also an argument could be given why muon beams are ideal candidates; at first glance, the fact that the muons are unstable and decay within less than a 1000 turns does not look extremely ideal for acceleration and collisions; how should/could the energies far in excess of electron-positron colliders be attained before the muons decay? Perhaps at least a reference can be given.

Various acceleration schemes are described in references 4-9 which are cited at the beginning of the paragraph.

In Equation (1), the emittance seems to have units of m^2/s, while in Eq. (5) and later in the text the unit is m or mm. I think there might be a factor "c "or perhaps "beta c" missing in Eq. (1).

A factor 1/c was added to the equation.

In line 157-159, "excluding particles … in higher amplitude bins … results in a distribution that, in the core of the beam, is independent of scraping effects and aberrations…" - is this necessarily correct? For example chromatic aberrations could well affect the transverse core of the beam, for example, unless there also is a cut in momentum deviation.

The text was modified to read "spherical aberrations".

Line 173, Eq. (7) is not common for accelerator physics and not easy to understand. Perhaps one more sentence of explanation would help? In addition, could one alternatively have chosen d=5 if the momentum deviation had been included as fifth dimension ?

The text was clarified.

Concerning the concept of a 5th dimension, this paper discusses only the transverse phase space as defined on line 109. Study of longitudinal phase space (momentum or energy deviation and time) is a separate analysis that is not considered in this paper.

Line 191, "4T" ->"4 T", with a blank.

The correction was adopted.

Line 233, perhaps the experimental value of beta_\perp could be given/recalled here.

The text was adapted to give the nominal beta_\perp in the experiment.

Line 275 ".. in a muon collider collective effects become significant only at very low longitudinal emittance [50]" - I suspect this statement refers only to space charge effects, discussed in the reference, and not to any other type of collective effects, e.g. possible ionization-related two-stream instabilities (hosing, self modulation, ionization-electron-driven beam breakup type of effects). The density of the ionizing material is high and the beam energy low, and the final muon beam will be bright. For example, long high-energy proton bunches suffer self modulation when passing through a plasma… could something similar happen here? Perhaps "collective effects" should be replaced by "space-charge effects", pending further studies.

The correction was adopted.

Line 328, should the term "normalised emittance" be defined?

Normalised RMS emittance is defined in equation 1.

Page 7 and figure 3, the "scraping" effect could be explained. What is it? Are muons lost or scattered when they hit an aperture? Could some of them be scattered towards the core, possibly resulting in a fake ionization cooling signal?

The text was modified to explain scraping and highlight the higher atomic number of the aperture material, such that the beam tends to heat in this region.

Line 479: The ingredients of the simulation for Fig. 4 could be indicated or a reference be given.

The simulation is described and referenced in the text on line 276-281. The text was updated to highlight this.